ADAPTIVE MEMORY PROGRAMMING : A UNIFIED VIEW OF METAHEURISTICS

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CONTENT

Meta-heuristics

Confuse situation due to multiple contributions & evolutions:

Multi

grasp

Algorithms

Genetic

Search

Adaptive

Scatter

Ant

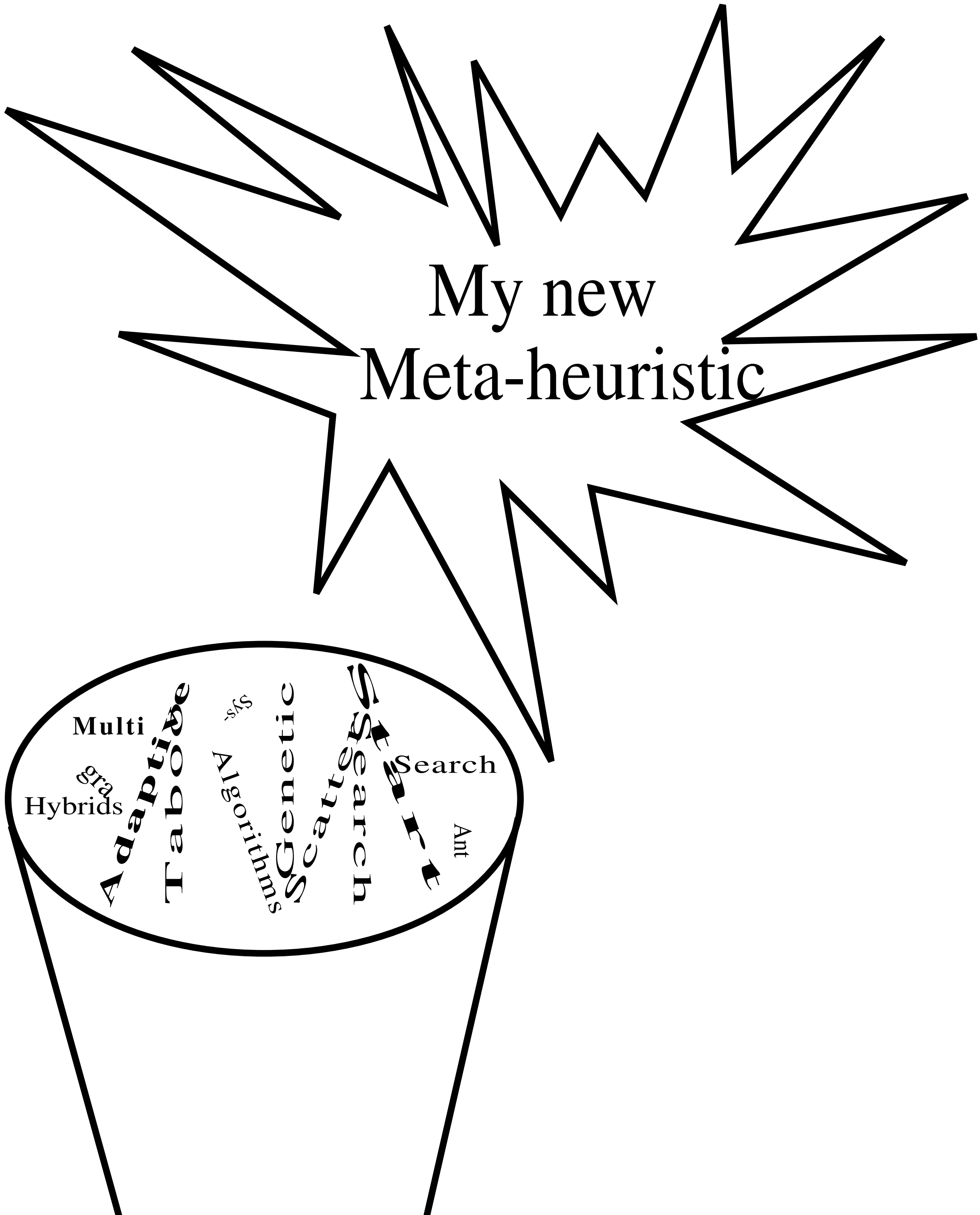
Hybrids

System

Start

Taboo

Search


My new
Meta-heuristic
Multi
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Hybrids
Adaptive
Taboo
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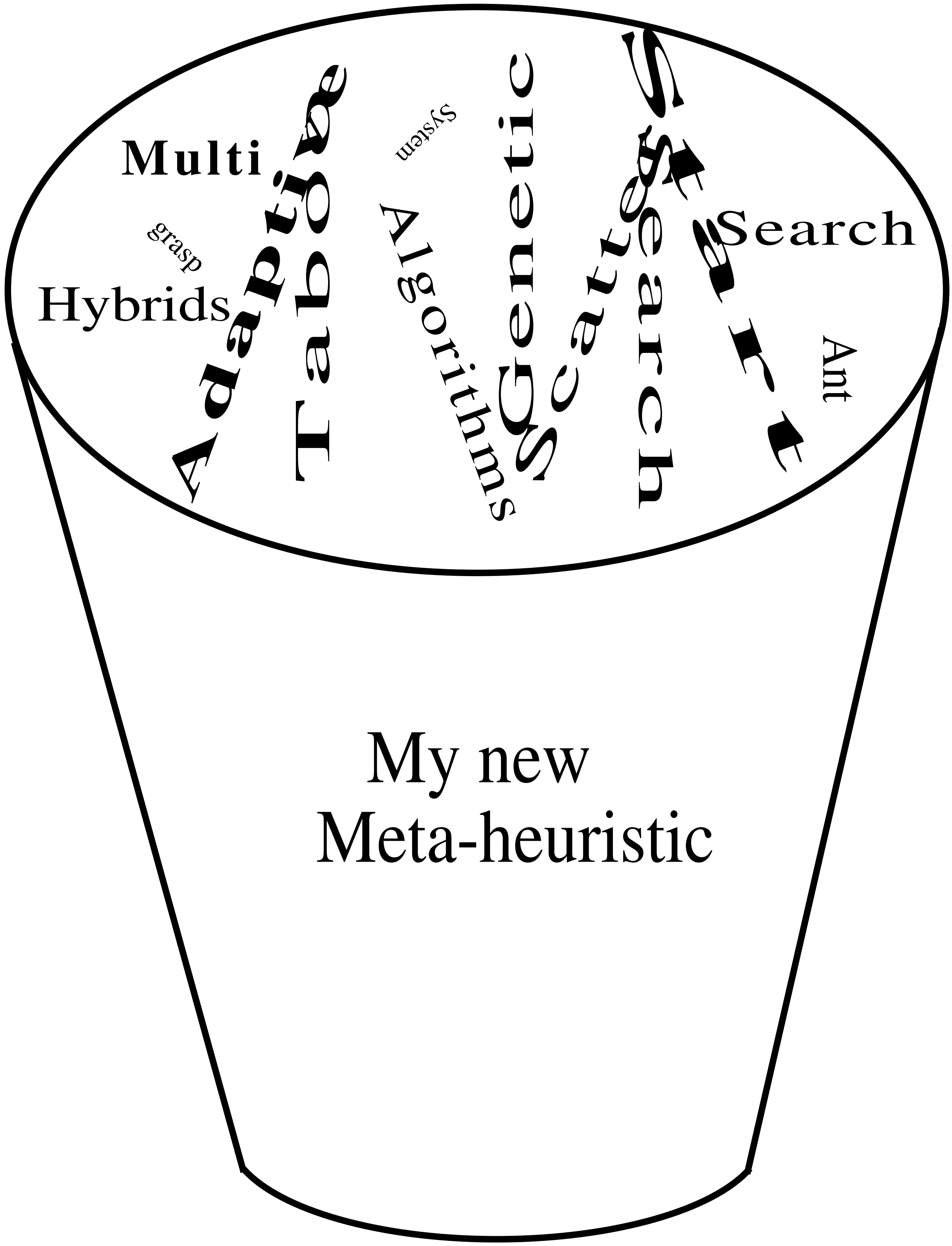
Algorithms
Genetic
Scatter
Search
Start
Search
Ant


Multi
grasp
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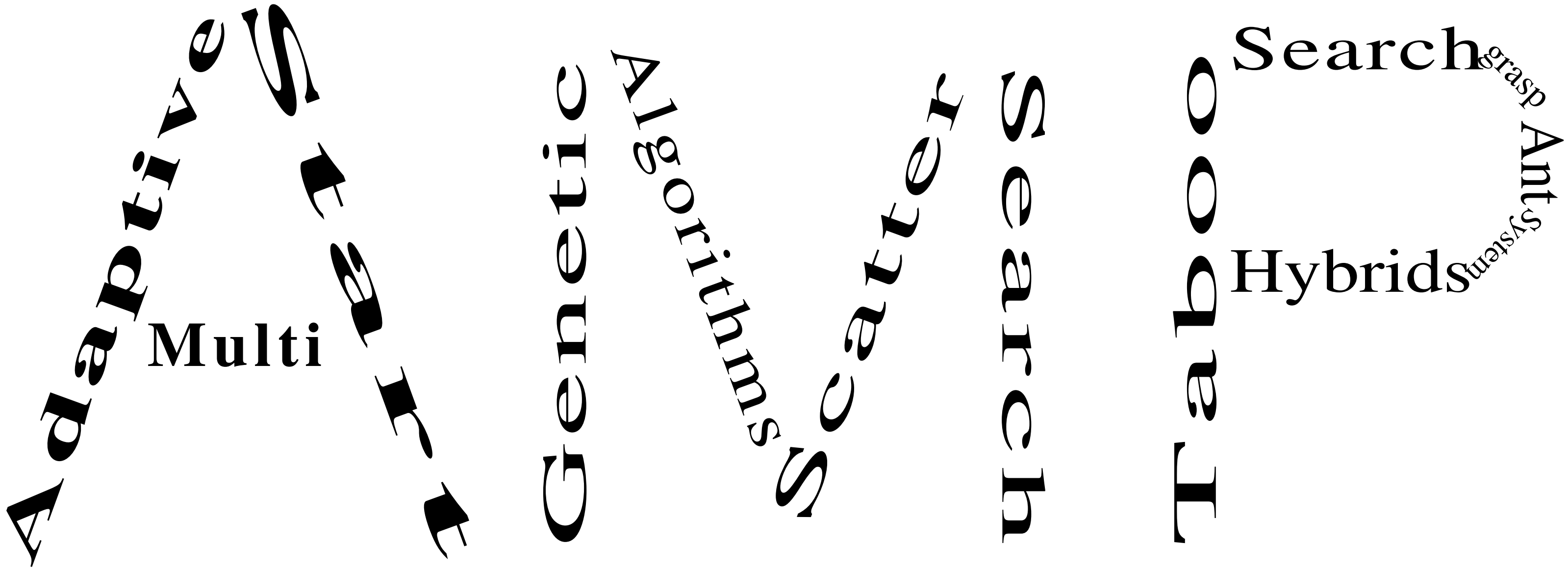
Tabu
System
Algorithms
Genetic
Scatter
Search
Search
Ant
My new
Meta-heuristic

EVOLVING TOOL BOX



META-HEURISTIC WITH MEMORY I
GENETIC ALGORITHMS
HOLLAND (1975)

Metaphor: Evolution of species sexually reproducing.

Basic genetic algorithm:

Choose a coding scheme for a solution

Generate a population of 0-1 vectors

While a stopping criterion is not met repeat:

- Select pairs of parent vectors from the population
- Apply cross-over to produce offspring vectors
- Apply mutation to each offspring
- Evaluate offspring
- Insert offspring in the population
- Eventually, remove individuals from the population with a culling operator

Evolution of genetic algorithms

0-1 vector coding

$\forall$ 0-1 vector corresponds a solution

global optimum have a 0-1 coding

Performance depends on coding

$\rightarrow$ natural coding (Davis, 1989, Eschelman & Schaffer, 1992)

Standard cross-over operators

1 point, 2 points, uniform

$\rightarrow$ specialized operators (Davis, 1989, Wright, 1990, Bäck et al., 1991)

2 parents cross-over

$\rightarrow$ multiple parents operator (Eiben et al., 1994, Mühlenbein & Voigt, 1996)

Mutation

random flipping of bits

$\rightarrow$ local search, repair operator (Mühlenbein et al., 1988, Ulder et al., 1991)

Culling $\rightarrow$ Strategies for subdividing population (Mühlenbein & Schlierkamp-Voosen, 1994)

NEW GENETIC ALGORITHMS

Generate a population of solutions

While a stopping criterion is not met, repeat:

Generate a new solution by combining a number of solutions from the population

Improve new solution with a local search

Update the population

NEW GENETIC ALGORITHMS

Initialize memory
Generate a population of solutions

While a stopping criterion is not met, repeat:

Build a provisory solution
Generate a new solution by combining a number of solutions from the population

Improve provisory solution
Improve the new solution with a local search

Update memory
Update the population

META-HEURISTIC WITH MEMORY II
SCATTER SEARCH
GLOVER (1977)

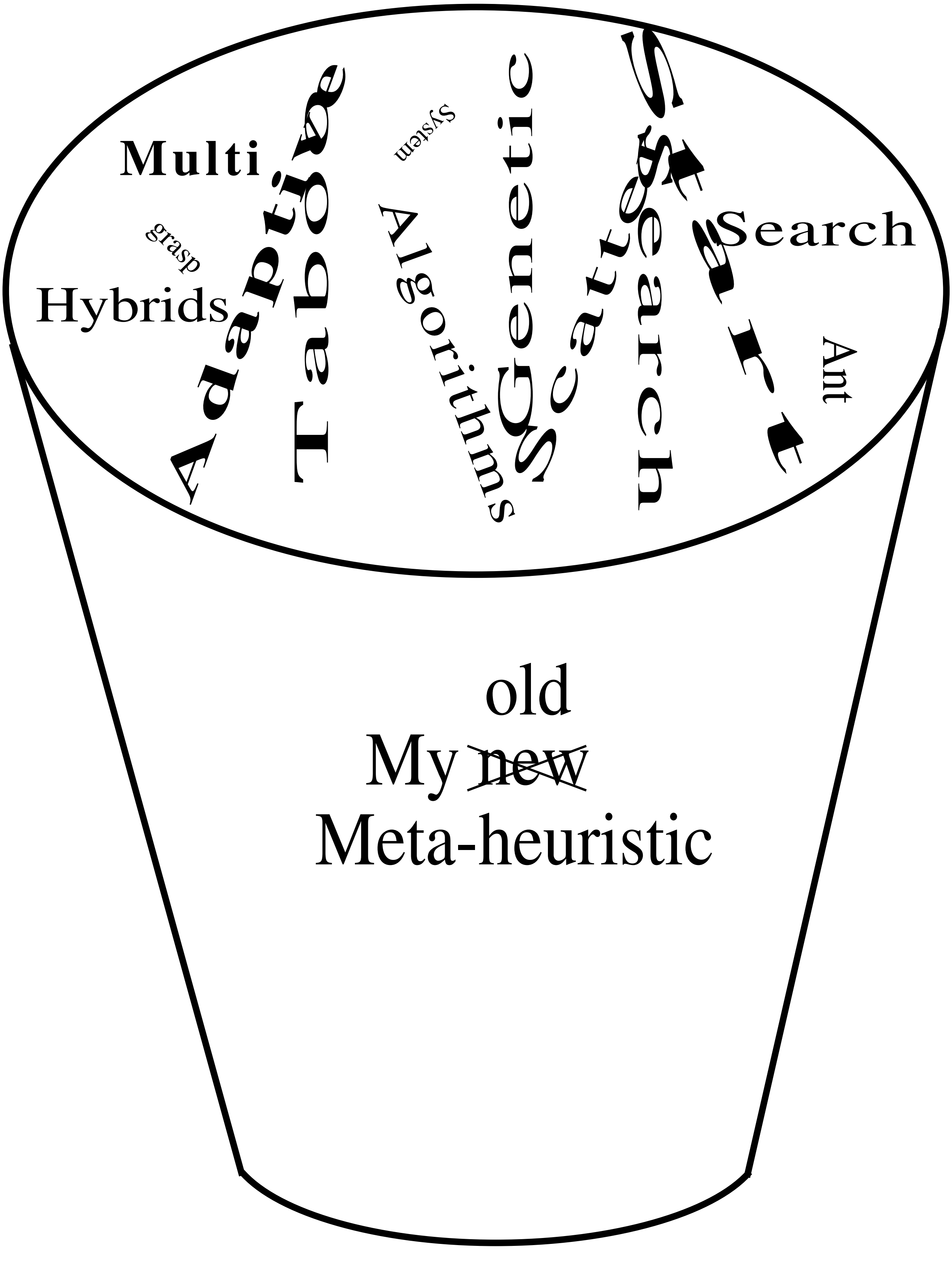
Basic scatter search (Integer linear programming)

Generate a population of integer vectors

While a stopping criterion is not met, repeat:

- Select a number of vectors from the population
- Build a new vector by linear combination of the selected ones
- Build a new solution by applying a repair operator that projects the new vector in feasible space
- Insert new solution into the population
- Purge population

GLOVER'S PRESENTATION OF SCATTER SEARCH:



NEW SCATTER SEARCH

→ Extension for dealing with problems other than ILP

Initialize memory

Generate a population of solutions

While a stopping criterion is not met, repeat:

Build a provisory solution

Build a new solution from a number of solutions selected in the population

Improve provisory solution

Improve provisory solution with a local search

Update memory

Insert the improved solution in the population and purge the population

Meta-heuristics with memory III: Taboo search Glover (1986)

Metaphor: Imitating human being building a solution to a combinatorial optimization problem

Basic taboo search:

Build an initial solution s_0; set $s^* \leftarrow s_0$; $k \leftarrow 0$.

While a stopping criterion is not met, repeat:

In the neighbourhood of s_k, SELECT a solution s_{k+1} that is not forbidden by taboo conditions

If s_{k+1} better than s^*, set $s^* \leftarrow s_{k+1}$.

Update taboo list.

TABOO SEARCH : OTHER COMPONENTS.

Intensification

Come back to best solution

Come back to (neighbours of) good solutions

Shorten taboo list

Candidate list

Proximate optimality principle

Diversification

Jump in solution space

Restart

Build new solutions using additional memories

Enlarge taboo list

Strategic oscillation

Path relinking

ADVANCED TABOO SEARCH

.

Initialize memory

Initialize short term and long term memories.

While a stopping criterion is not met, repeat:

Build a provisory solution

Generate a new initial solution with the help of long term memory and intensification/diversification policies

Improve provisory solution

Improve provisory solution with a basic taboo search (parameters given by intensification/diversification)

Update memory

While running the basic taboo search, collect informations and update memories

Meta-heuristics with memory IV : Ant systems Colorni, Dorigo, Maniezzo (1990)

Metaphor : Real ants are optimizing their search for food by depositing chemical traces (pheromone trails) on the floor.

Basic ant algorithm :

Initialize pheromone trails

While a stopping criterion is not met, repeat in parallel for p ants:

Probabilistically build a new solution, using a partial evaluation function and pheromone trails values

Update pheromone trails

New ant systems

→ Acceleration of convergence with local search, intensification

→ Diversification in case of early convergence

Initialize memory

Initialize pheromone trail matrix

While a stopping criterion is not met, repeat:

Build a provisory solution

Probabilistically build a new solution, using a partial evaluation function and pheromone trails values

Improve provisory solution

Improve provisory solution with a local search

Update memory

Update pheromone trail, taking diversification and intensification policies into account

Adaptive Memory Programming (AMP)

Initialize memory

While a stopping criterion is not met, repeat:

Build a provisory solution (using memory)

Improve provisory solution

Update memory

Other related meta-heuristics: Grasp, Adaptive multi start

$\rightarrow$ Simple scheme but large choices:

Memory type, implementation and update (population, statistics, solution parts, …)

Provisory solution construction (specialized cross-over, "average" solution, vocabulary building, intensification diversification, jumps …)

Improving procedure (descent method, taboo search, simulated annealing, noisy search, alternated neighbourhoods, AMP …)

Strength of AMP

Intensify search with improving procedure

Diversify search through provisory solution building procedure

Find structure of good solutions

Efficient for real instances

Dynamic problems

Parallelization

Applications of AMP I: Vehicle routing

Problem: (minimum length CVRP)

Given

n customers with weights

1 depot

1 vehicle of capacity Q

distances between customers and depot

Find

a set of tours for the vehicle, starting from and ending at the depot

Such that

the total weight of each tour is at most Q

the total length travelled by the vehicle is minimized

VEHICLE ROUTING: OTHER VARIANTS

Data

time window for each customer

service time for each customer

m vehicles

multiple depots

heterogeneous fleet of vehicles

penalties

Objective

minimize m

minimize longest tour or longest day

penalized objective

Constraints

maximum tour duration

maximum working hours per day

service during time windows

one tour per vehicle

AMPs for VRPs

Memory

Set of tours of the best solutions produced

Initialization: multiple short taboo searches

Provisory solution generation (vocabulary building)

Repeat, while an appropriate tour can be found:

Randomly select a tour from the memory, such that:

Probability of selection depends on the quality of the tour

The tour does not contain customers already belonging to selected tours

Improvement procedure

Complete provisory solution and improve it with a short taboo search

Taillard (1994, 1996), Rochat & Taillard (1995), Gendreau, Guertin, Potvin & Taillard (1996), Taillard, Laporte & Gendreau(1996), Badeau, Guertin, Gendreau, Potvin & Taillard (1997), Golden, Laporte, Taillard (1997), Taillard, Badeau, Gendreau, Guertin & Potvin (1997), …

GRAPH COLOURING

Given

A graph (set of vertices and edges between vertices)

A number of colours *m*

Find

a partition (colouring) of the vertices into *m* subsets

Such that

the number of edges connecting two edges belonging to the same subset is minimized

AMP FOR GRAPH COLOURING I

Zufferey, 1998

Memory

Set of subsets of the best solutions produced

Initialization: multiple greedy descents

Provisory solution generation

Repeat, while an appropriate subset can be found and less than *m'* subsets have been chosen:

- Randomly select a subset from the memory, such that:
  - Probability of selection depends on subset quality
  - The subset does not contain vertices already belonging to selected subset

Complete solution with a greedy descent

Improvement procedure

Improve provisory solution with a short taboo search (Tabucol, Hertz & de Werra, 1987)

AMP FOR GRAPH COLOURING II, III

Costa, Hertz & Dubuis, 1995, Fleurent & Ferland, 1996

Leading idea: genetic/taboo or descent hybrid

Memory

Population of best colouring found.

Provisory solution

Select 2 solutions from the population, choose the colour of each vertex from those of the selected solutions

Improvement procedure

Greedy local search (CHD)

Tabucol (FF)

QUADRATIC ASSIGNMENT PROBLEM

Given

Two $n \times n$ matrices $\boldsymbol{A}$ and $\boldsymbol{B}$

Find

A permutation π

Such that

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} b_{\pi_i \pi_j} \quad \text{is minimized}$$

AMP FOR QAP I

Fleurent & Ferland (1994)

Leading idea: genetic/taboo search hybrid

Memory

Population of 100 solutions

Provisory solution

Select two solutions from the population, apply a cross-over operator specialized for permutations (Tate & Smith, 1995)

Improvement procedure

Basic taboo search (Taillard, 1991)

AMP FOR QAP II

Cung, Mautor, Michelon & Tavares, 1997

Leading idea: Scatter search/taboo search hybrid

Memory

Population of solutions

Provisory solution

Select a number of solutions (vectors) from the population

Compute a linear combination of the selected vectors

Find the permutation the closest from linear combination

Improvement procedure

Taboo search, with intensification and diversification strategies

AMP FOR QAP III, IV (FANT, MAXMIN ANT SYSTEM)

Taillard & Gambardella, 1997

Stútzle & Hoos, 1997

Leading idea: Ant systems

Memory

Statistics matrix recording the frequency of the position of each element in the permutations generated

Intensification and diversification mechanisms

Provisory solution

Randomly choose the elements of the permutation with a probability proportional to the statistics matrix

Improvement procedure

Greedy local search or taboo search

AMP for QAP V
(HAS-QAP)

Gambardella, Taillard & Dorigo, 1997

Leading idea: Ant system/local search hybrid

Memory

Population of solutions (1 per ant), statistics matrix (pheromone trail). Intensification and diversification mechanisms

Provisory solution

Apply a number of random moves to each solution of the population. Probability of choosing a move proportional to statistics matrix entries

Improvement procedure

Fast greedy local search

AMP : The future

Memory

Considering other learning principles

Neural networks

Reinforcement learning

Provisory solution building : natural intelligence !

Improvement procedure

— Advances in local search (Aarts & Lenstra, 1997, Weinberger, 1990, Stadler, 1995, Angel & Zissimopoulos, 1998)

— New neighbourhoods

— Problem decomposition

Dynamic problems

Adaptation to moving environment

Fast response time

Efficiently use previous work

May propose many solutions simultaneously

Parallel implementations

Parallel provisory solution construction and improvement

NEW NEIGHBOURHOOD

∃ descent algorithm (with neighbourhood size $O(pn)$) often producing better solutions than taboo searches, variable neighbourhood search, genetic algorithms to p-median, sum-of-squares clustering or multi-source Weber instances (Taillard, 1996).

Combined with FANT: Reach or improve best solutions known to 100 hard instances due to Rolland et al., 1996 (Taillard, 1998).

.

n	p	TS (Rolland et al., 1996)	HC (Rosing, 1998)	VNS (Mladenovic & Hansen, 1998)	FANT (Taillard, 1998)
200	10	49244	**48912**	**48912**	**48912**
	15	32024	**31153**	**31153**	**31153**
	20	23496	23475	**23323**	**23323**
300	10	83049	**82664**	83176	**82664**
	15	53729	**52772**	52865	**52772**
	20	39195	38761	**38244**	**38244**
400	10	128147	**123464**	123701	**123464**
	15	83983	80473	81892	**79872**
	20	59142	58459	59617	**58408**
500	10	152289	**150112**	150381	**150112**
	15	98391	**97624**	98664	**97624**
	20	73153	73153	74328	**72714**

SUB-PROBLEM OPTIMIZATION (LOPT)

Idea : Isolate part of a solution, optimize this part independently.

- → Highly efficient for optimizing large uniform problems
- → Parallel processing

Vehicle routing (Taillard, 1993)

Sub-problem definition : subset of tours

Optimization : basic taboo search

Typically reaches solutions at 1% above best known

Centroid clustering (Taillard, 1996)

Sub-problem definition : subset of centres (+entities)

Optimization : truncated descent method with new neighbourhood

Low complexity (hardest work : find the subset of closest centres)

Typically reaches solutions below 1% above best known

Similar idea : VNDS (Hansen & Mladenovic, 1998)

IS THE CURRENT PASSING?