Sweep and Path Relinking Procedures for the bi-objective QAP

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Quadratic Assignment Problem (QAP)

Mono-objective QAP :

Given two $n \times n$ matrices $F = (f_{ij})$ (flows between facilities i and j) and $D = (d_{rs})$ (distances between locations r and s), find a permutation π of n element (π_i being the location of facility i) minimizing :

$$\sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij} \cdot d_{\pi_i \pi_j}$$

Bi-Objective QAP :

Typical application : take (partially) into account the size of the facilities

Two flow matrices F^1 (e.g. interactions) and F^2 (e.g. inverse of the size of facilities) and two distance matrices D^1 (e.g. actual distance) and D^2 (e.g. inverse of distance).

Special case : $F^1 = F^2$

Most of the literature treat this special case.

Advantage : Scalarization of the objectives can be treated as mono-objective QAP

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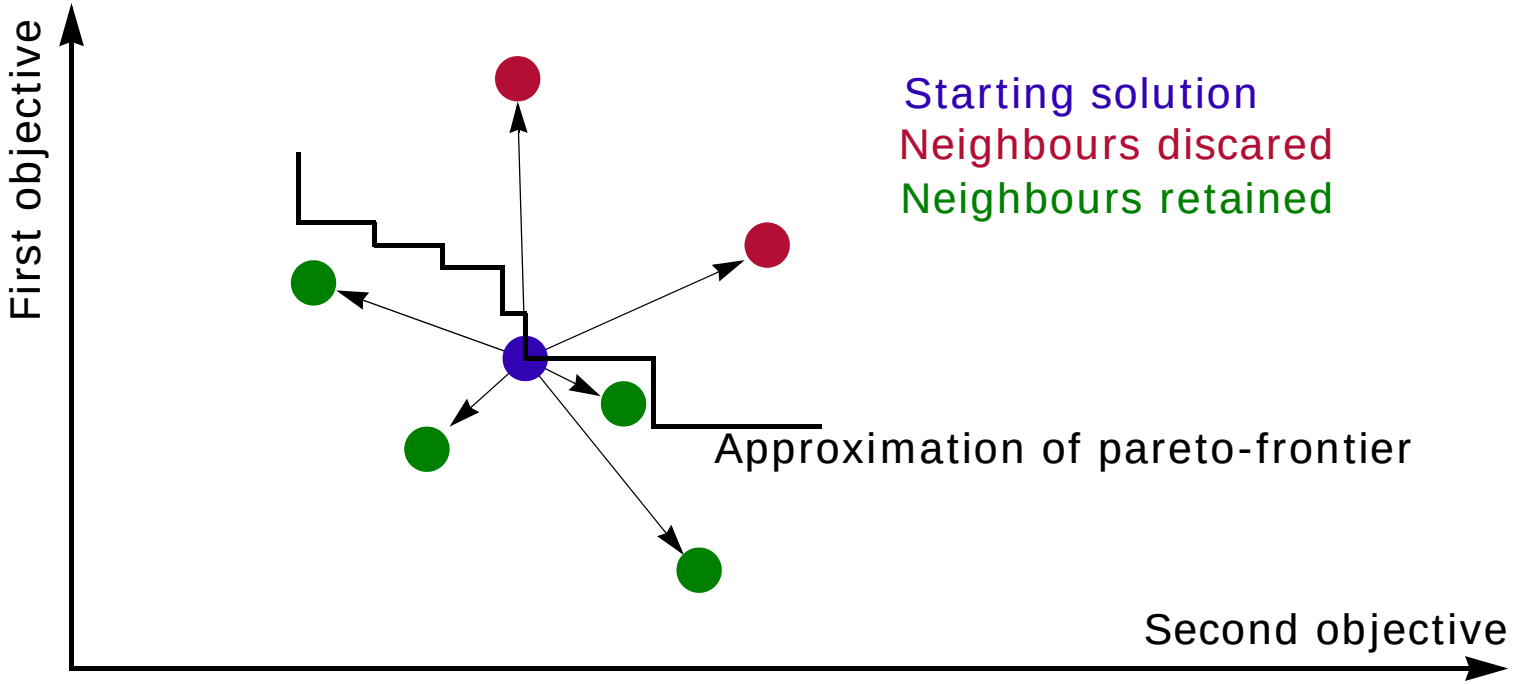
Existing method : Pareto Local Search (PLS)

Luis Paquete, Thomas Stützle, A study of stochastic local search algorithms for the biobjective QAP with correlated flow matrices, *European Journal of Operational Research* 169 (2006) 943–959

Local search neighbourhood :

Transposition of two elements of the permutation



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Existing method : Weighted Robust Taboo Search (W-RoTS)

É. D. Taillard, "Robust taboo search for the quadratic assignment problem", *Parallel Computing* 17, 1991, 443 - 455.

W-RoTS (Paquete & Stützle)

500 scalarizations of the objective function vector (with maximally dispersed weights)

Ro-TS is run for 100n iterations for each scalarization.

Goal : obtaining an very good approximation of pareto-frontier.

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Existing Method : Two-Phases Local Search (TPLS)

PLS characteristics

Approximation of pareto set concentrates on the centre

Points minimizing either objectives never found

Very short computational times for positively correlated matrices

Very bad approximation for positively correlated matrices

Good ratio quality of solutions/time for negatively correlated matrices

First Phase

Find solution s minimizing Objective 2 with mono-objective Ro-TS ($10n$ iterations)

Second Phase

For $\lambda = 1/n$ to 1 by step $1/n$

Starting with s, find a local optimum s' relatively to scalarized objective

λ Objective 1 + $(1 - \lambda)$ Objective 2

$s := s'$

D-TPLS : repeat with first phase minimizing Objective 1

PD-TPLS : Apply PLS to s'

TPD-TPLS : Ro_TS, all neighbours compared to pareto, local search : Ro-TS($n/3$)

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Sweep Robust Taboo Search

Parameters

I : Total number of iterations ($50n$, $200n$, ...)

λ_{init} : Initial value of λ (0.5 in all our experiments)

p : Percentage of iterations performed initially with λ_{init} (10% in all experiments)

S : Number of sweep (1 in all experiments)

First $p \cdot I$ iterations

Scalarized Ro-TS with λ_{init}

Next $(1-p)I$ iterations

At iteration i, scalarized Ro-TS with $\lambda = \frac{1}{2}\left(1 + sin\left(\frac{2\pi S i}{(1-p)I} + atan(\lambda_{init})\right)\right)$

Each neighbour solution is considered for entering pareto set

Hope

Find good compromise initially

Then insists on single first and second objectives

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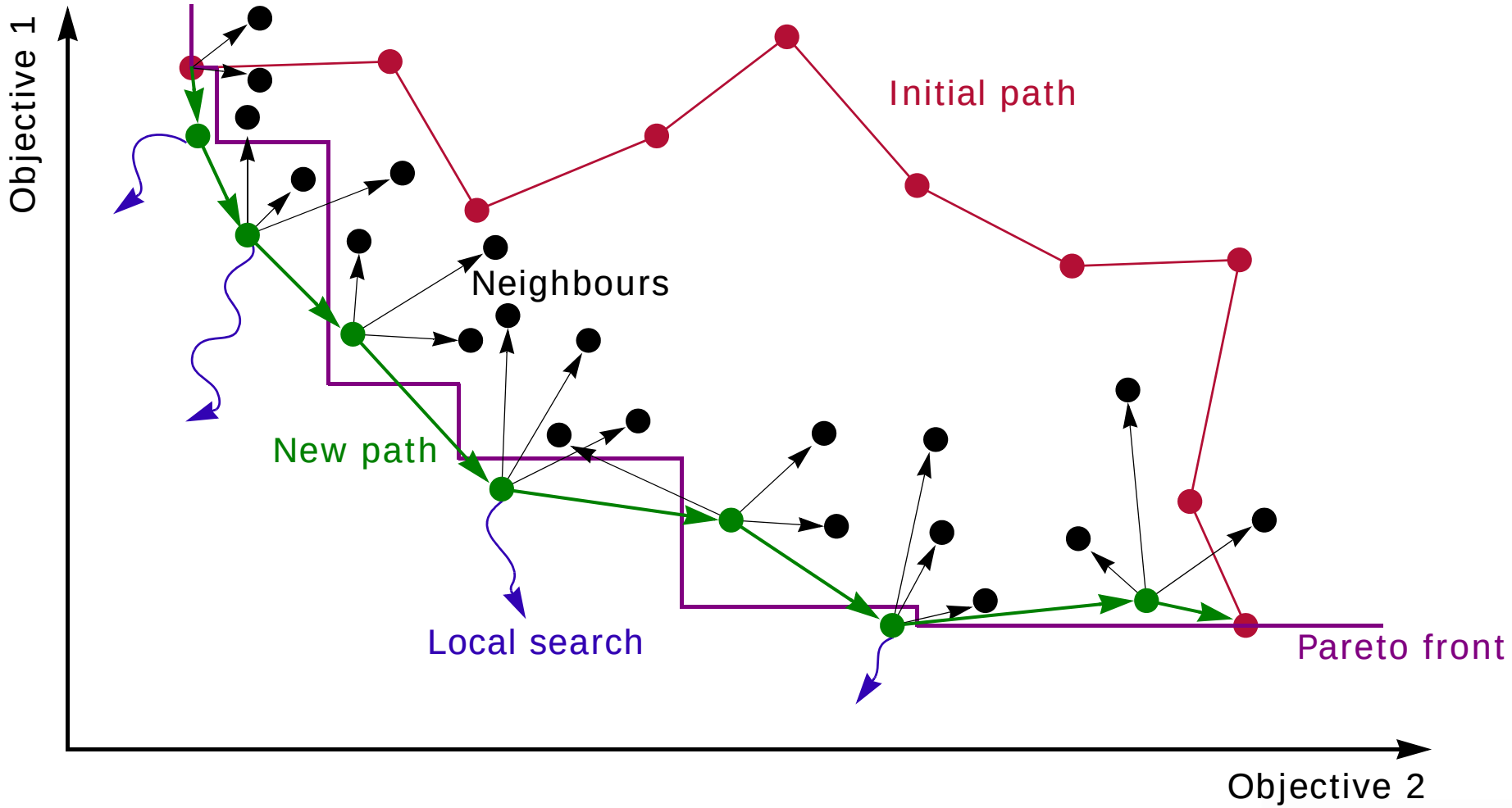
Path Re-linking

Strategy proposed by F. Glover in the context of Tabu search

Take 2 good solutions obtained during TS and re-link them with another path



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Observations on Path Re-linking for bi-QAP

Linking 2 solutions far from another often lead to paths containing dominated solutions

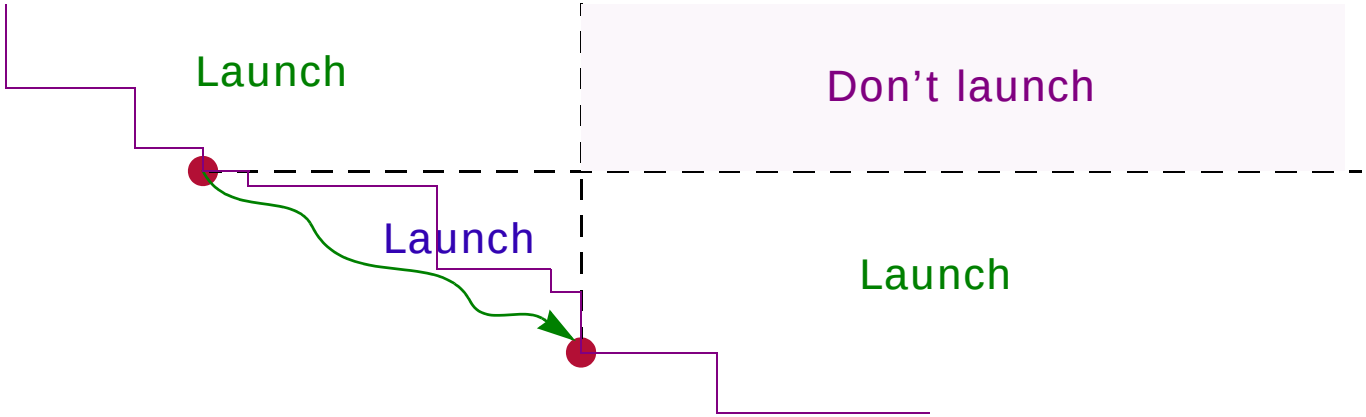
Re-link a given solution to *r* closest solutions found (*r* = 20)

Efficient solution set can be too limited or too large

Add dominated solutions (not tried)

Launching a local search each time pareto frontier improved may be to restrictive

Launch also for other solutions



Options tried :

Launch a 4*n* iterations Ro-TS only when pareto frontier improved. Long PR

Launch PLS if solution not strictly dominated either by starting or ending solution

Short PR

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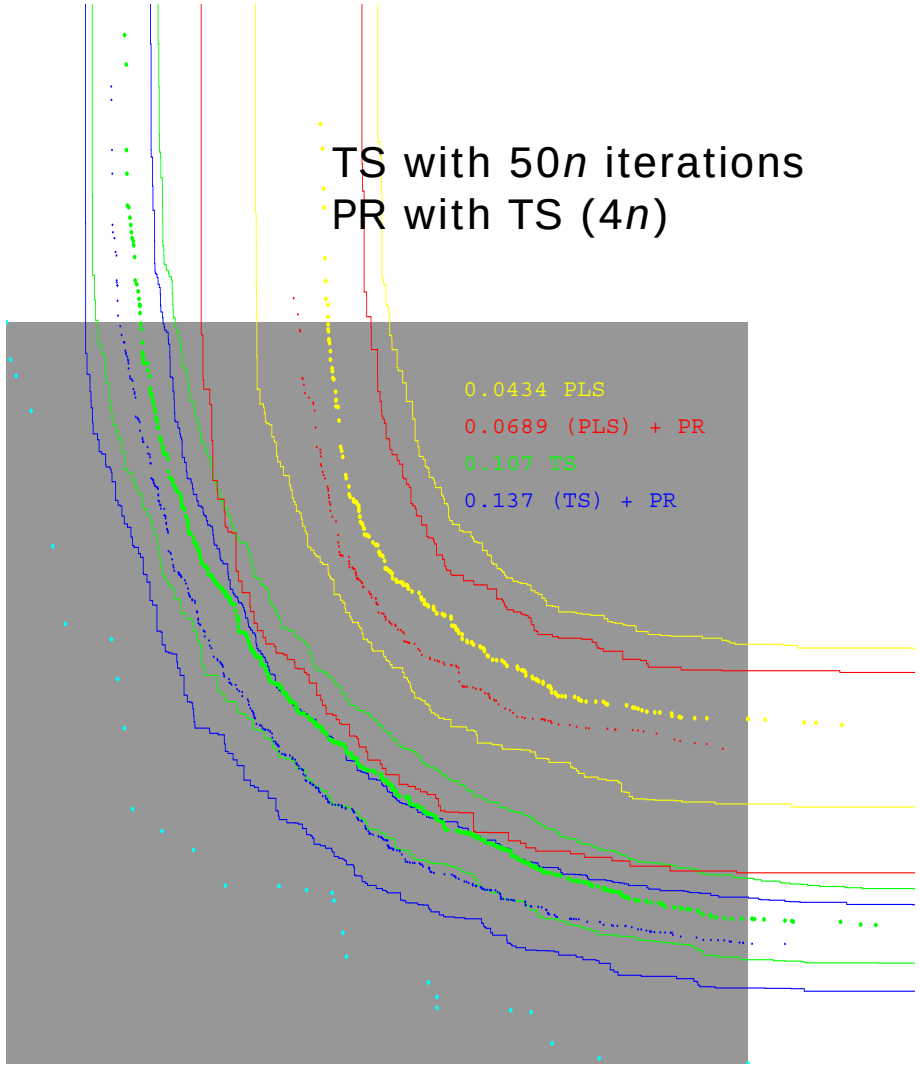
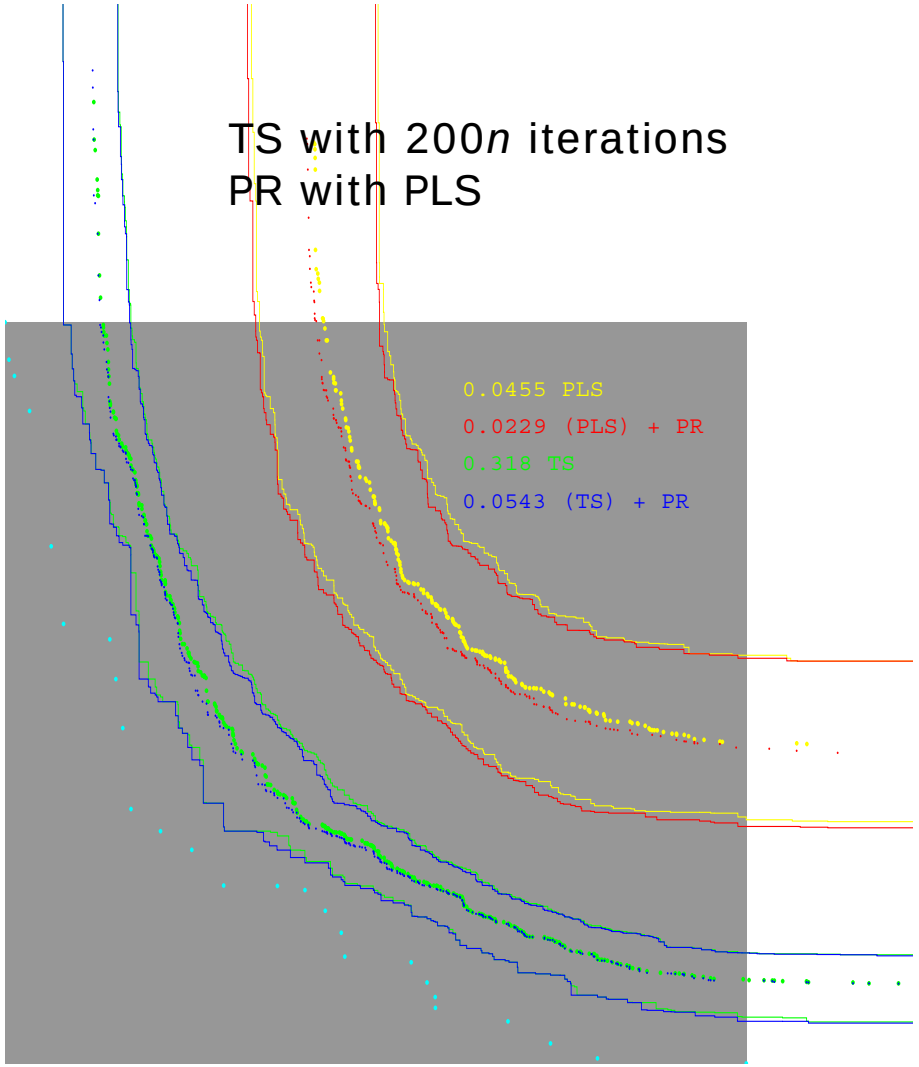
Numerical results

Correlated (0.5) uniform instance, n = 25





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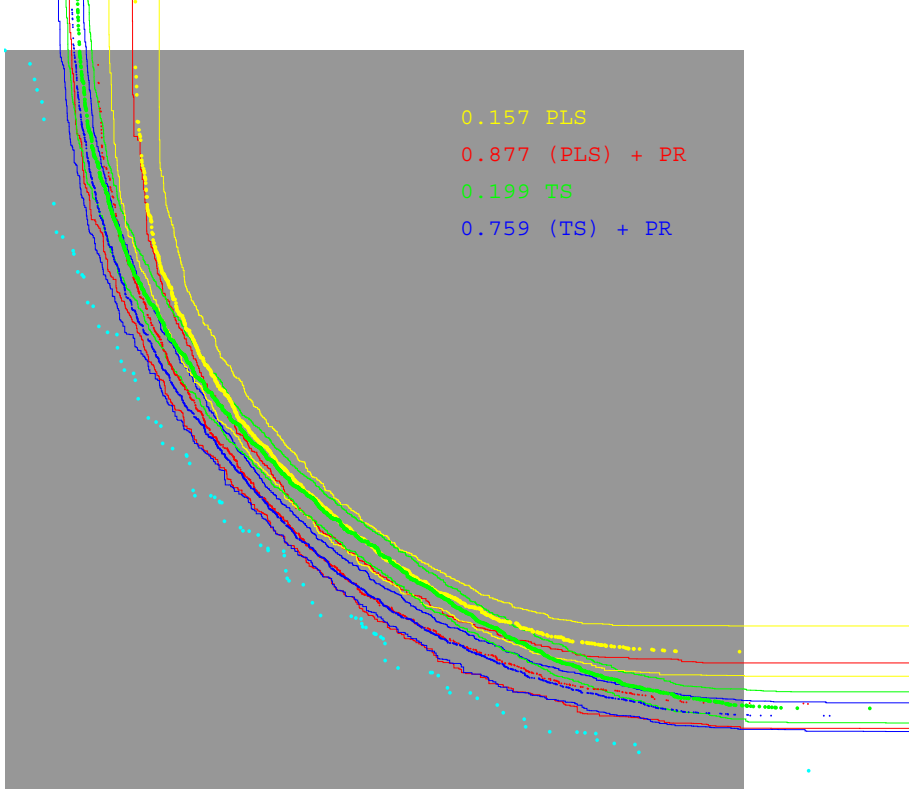
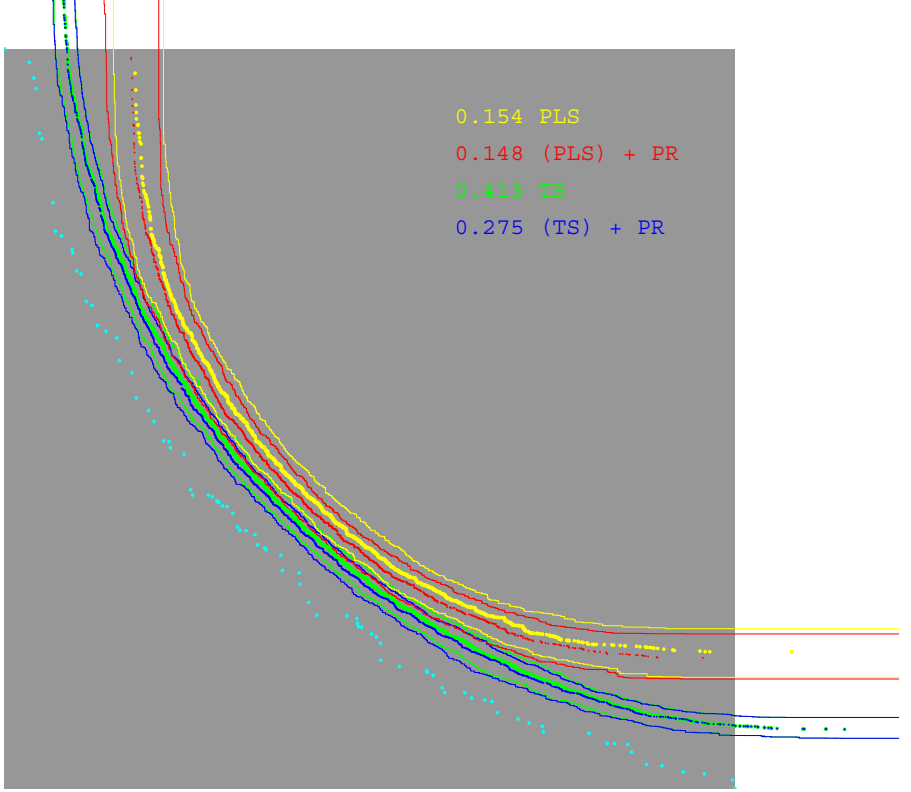
Numerical results

Uncorrelated uniform instance, n = 25

TS with 50*n* iterations
PR with TS (4*n*)



TS with 200*n* iterations
PR with PLS



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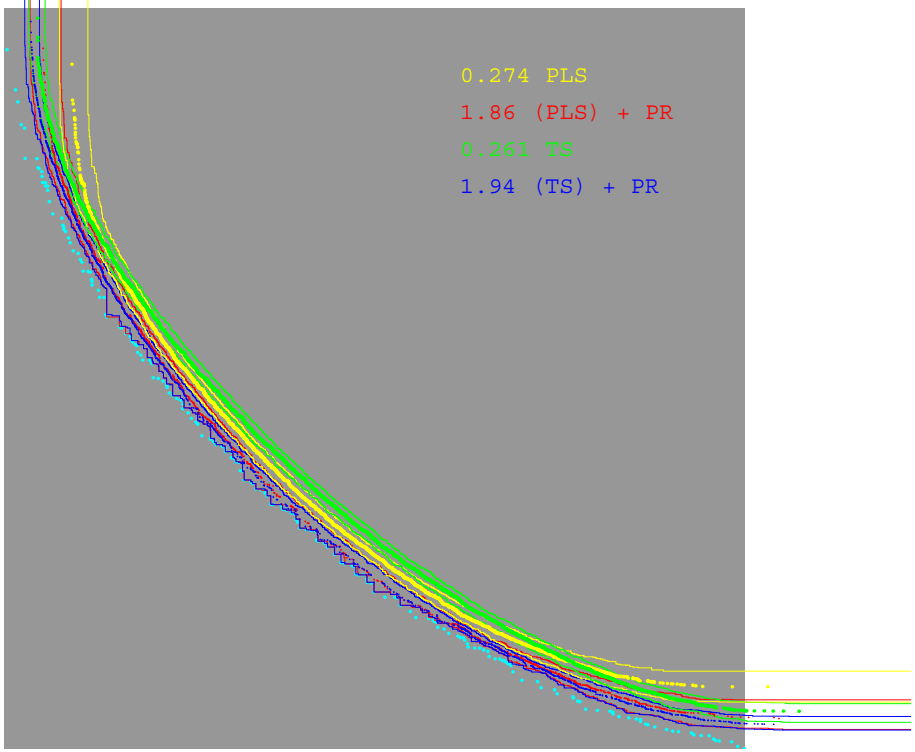
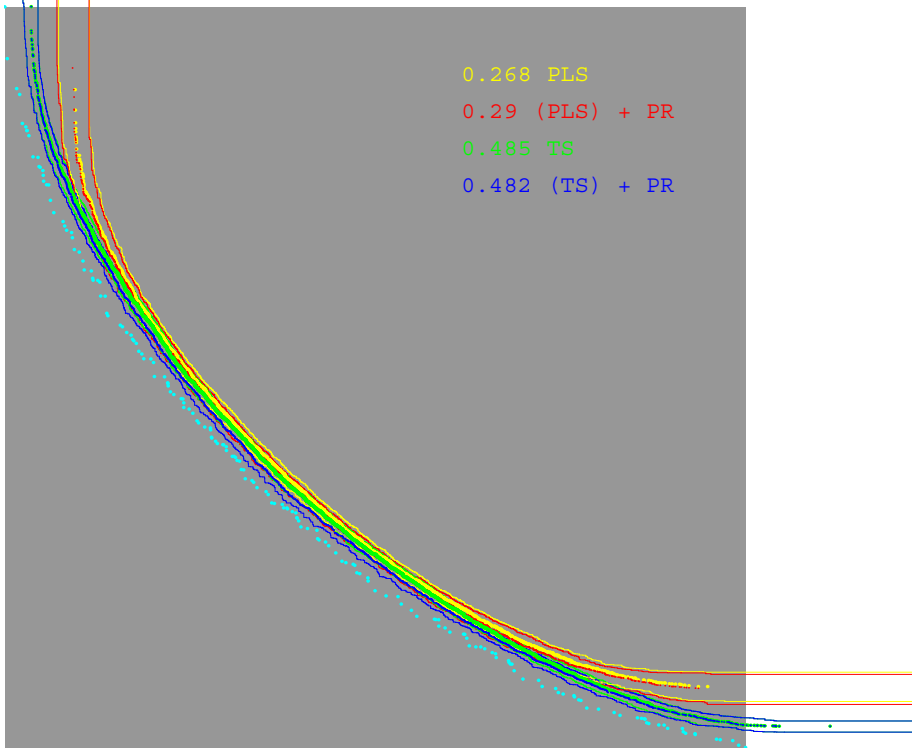
Numerical results

Correlated (-0.5) uniform instance, n = 25

TS with 50*n* iterations
PR with TS (4*n*)



TS with 200*n* iterations
PR with PLS



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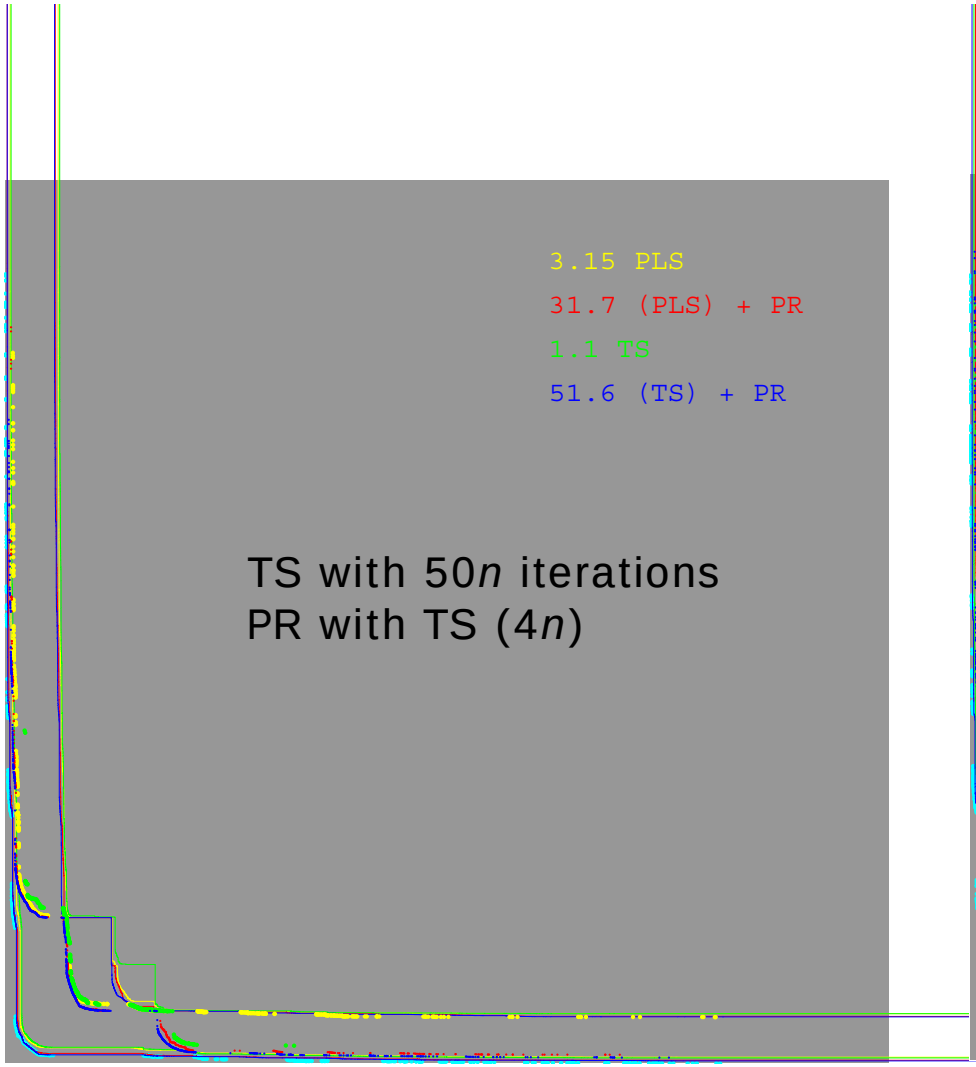
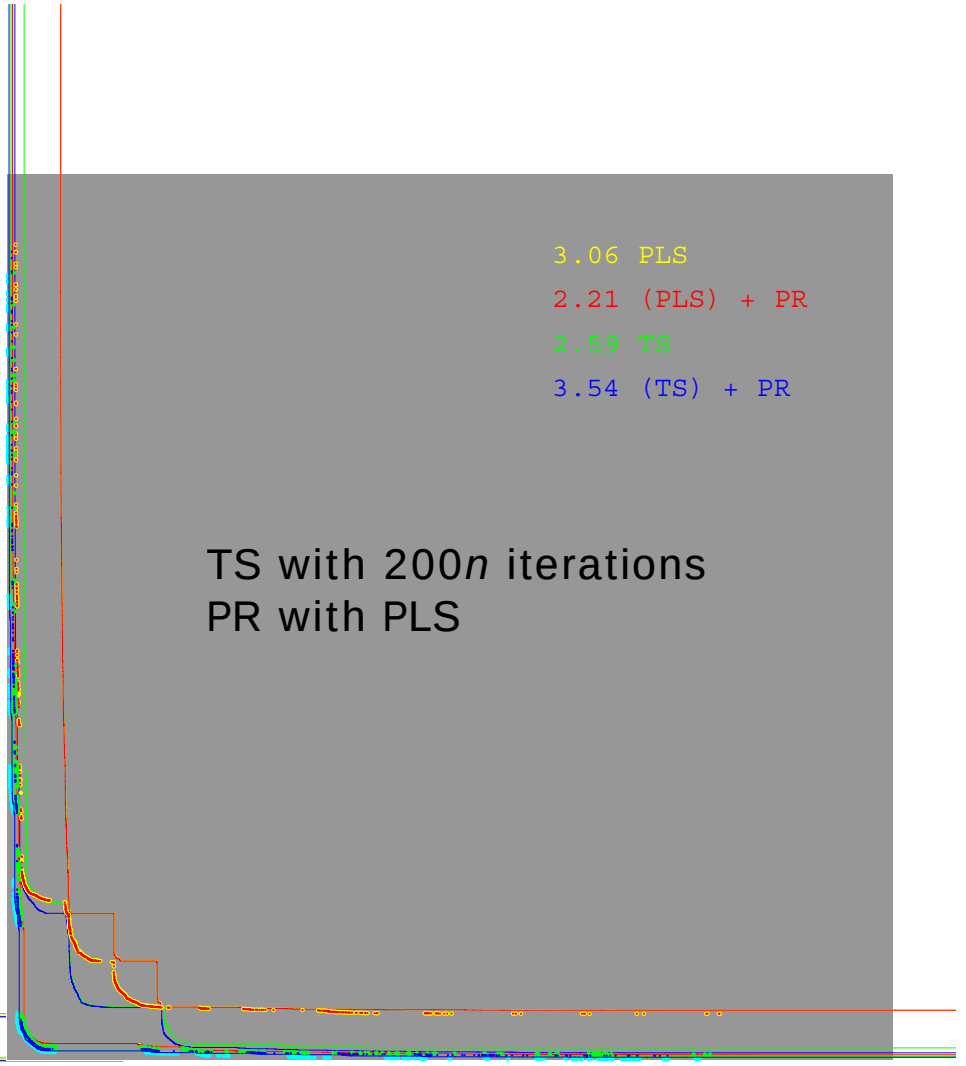
Numerical results

Uncorrelated, non uniform instance (tai45e01+tai45e02, n = 45)





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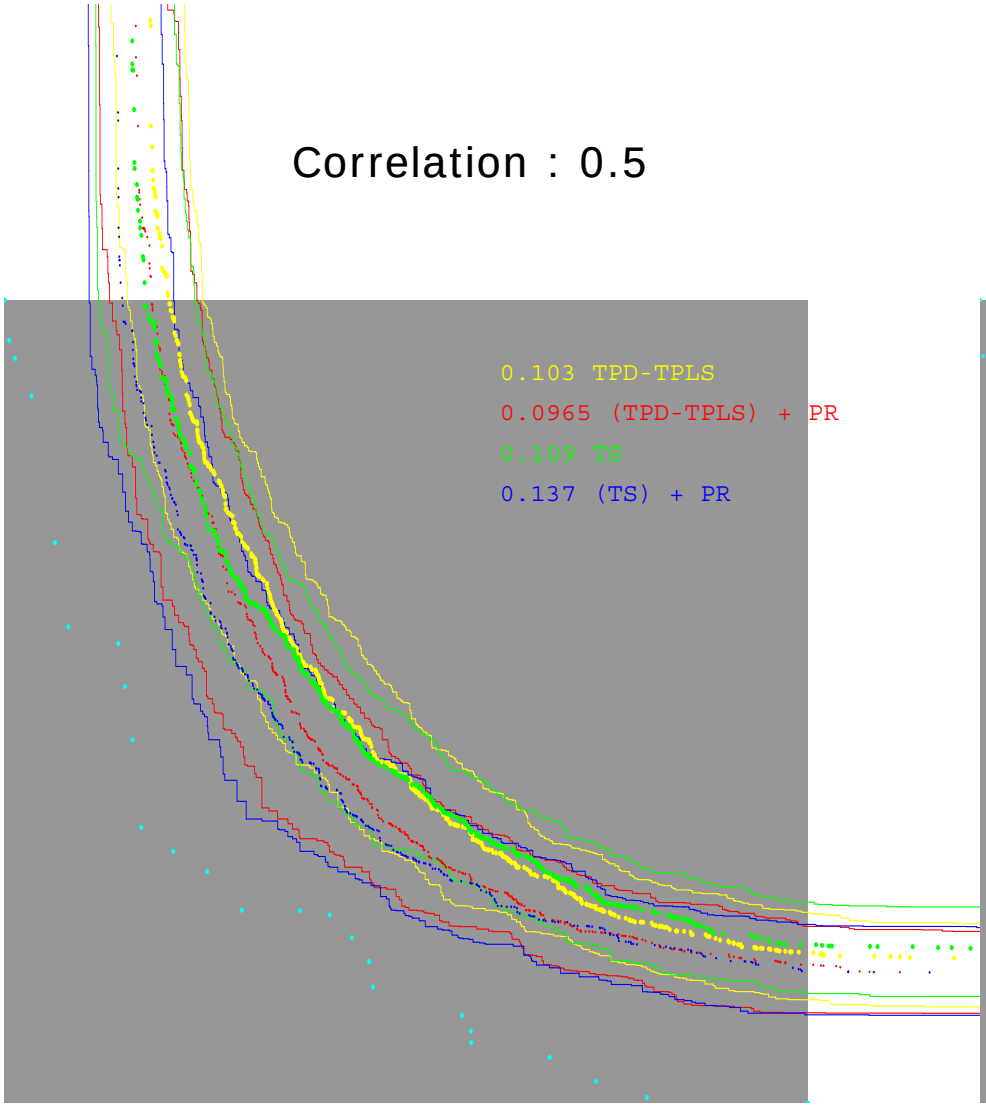
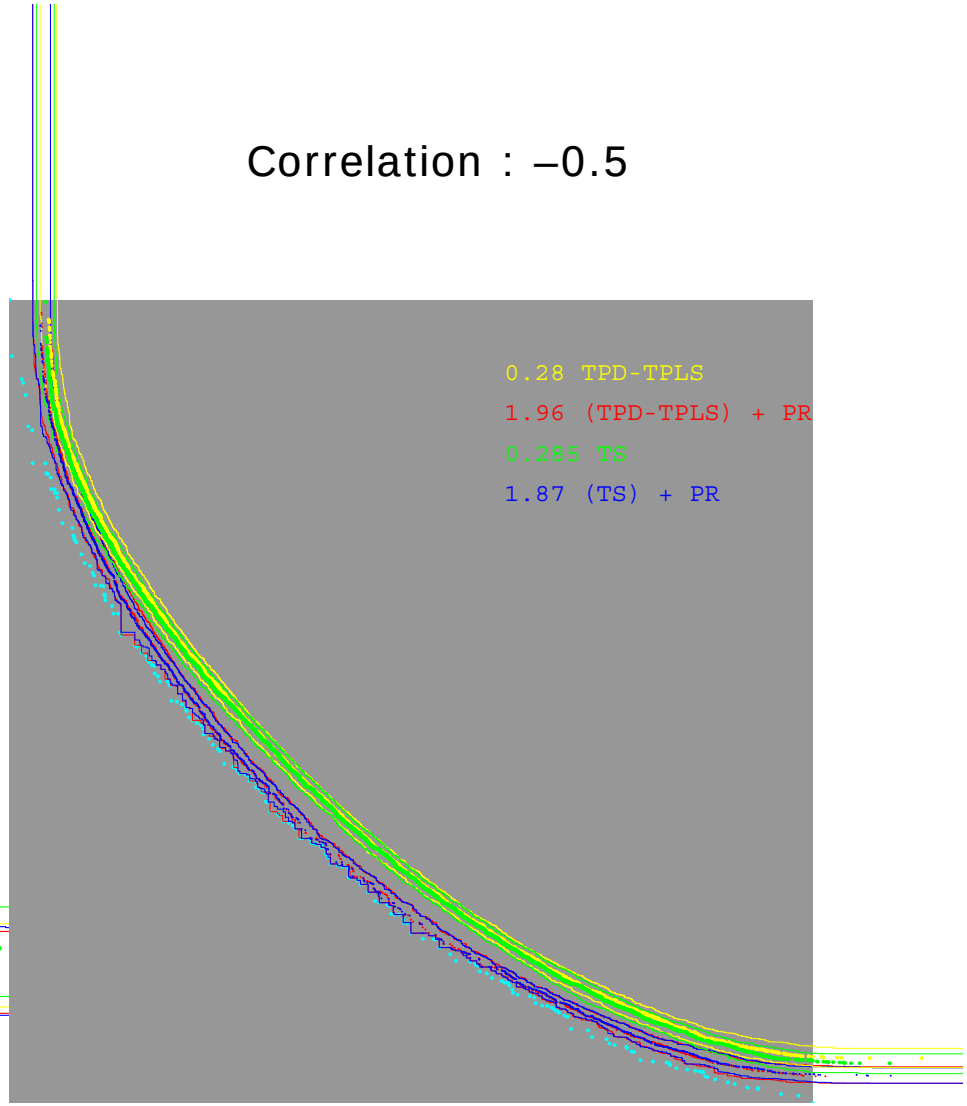
Numerical results : Comparison with TDP-TPLS

TS with 50*n* iterations, PR with TS(4*n*)

Correlation : 0.5



Correlation : –0.5



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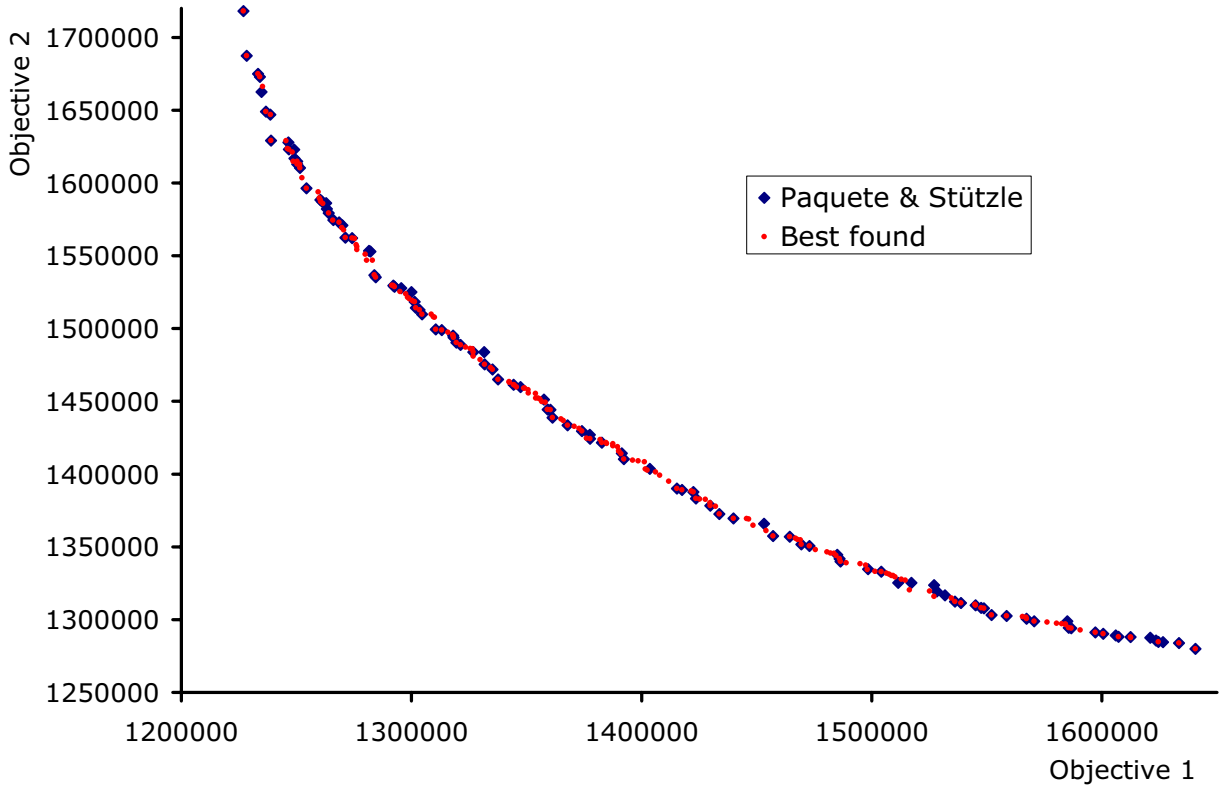
REFERENCE SET



Correlation : –0.5

P&S : 108 solutions
Best : 193 solutions

Lower computational effort

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CONCLUSIONS

Sweep Ro-TS very competitive regarding to other methods

Path Re-linking difficult to set-up

Simple PR implementation not so efficient for bi-QAP

Future work

- Experiments on more problem instances
- Reporting comparison metrics
- Exploitation of different parameter settings for Sweep Ro-TS
  - Number of sweep
  - Initial direction
  - Initial effort in that direction

- Other options for Path Re-linking

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