A Template for Low-Complexity POPMUSIC Implementation

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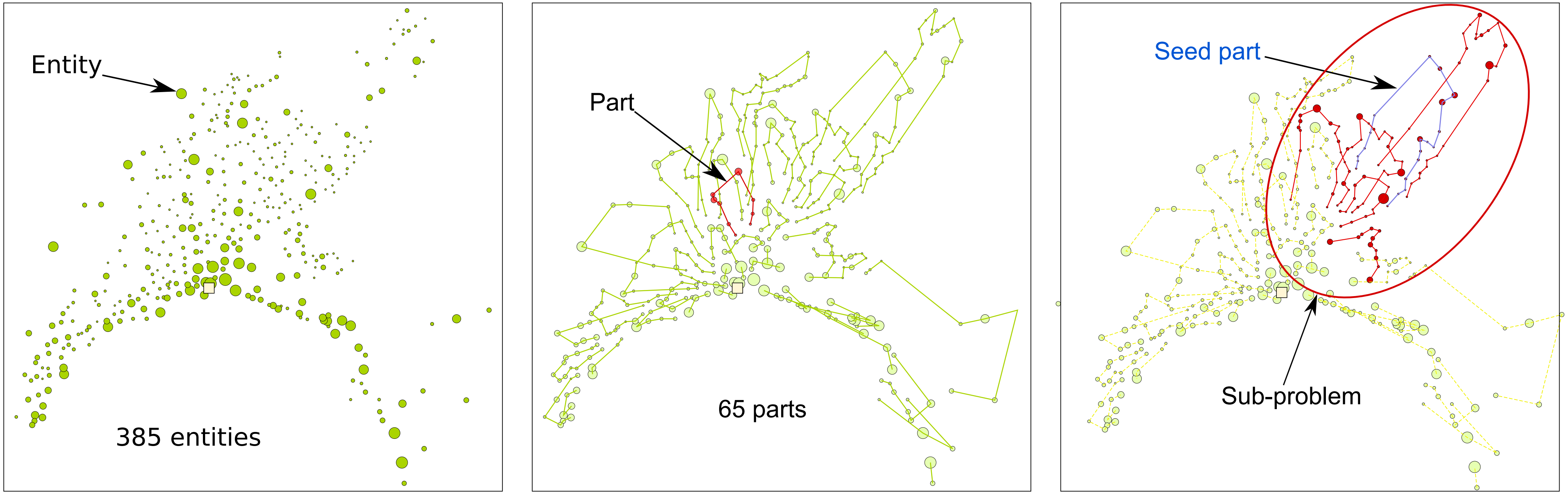
POPMUSIC General idea

POPMUSIC =

Partial OPtimization Metaheuristic Under Special Intensification Conditions

Special Intensification Conditions



A solution can be decomposed into somewhat independent parts

A subset of part (sub-problem) can be optimized almost independently

Pertinence of problem decomposition

Hypothesis

Large problem instances but moderate dimension

⇒ 2 elements close to a third one are also close

⇒ 2 elements far away cannot be both close to a third one

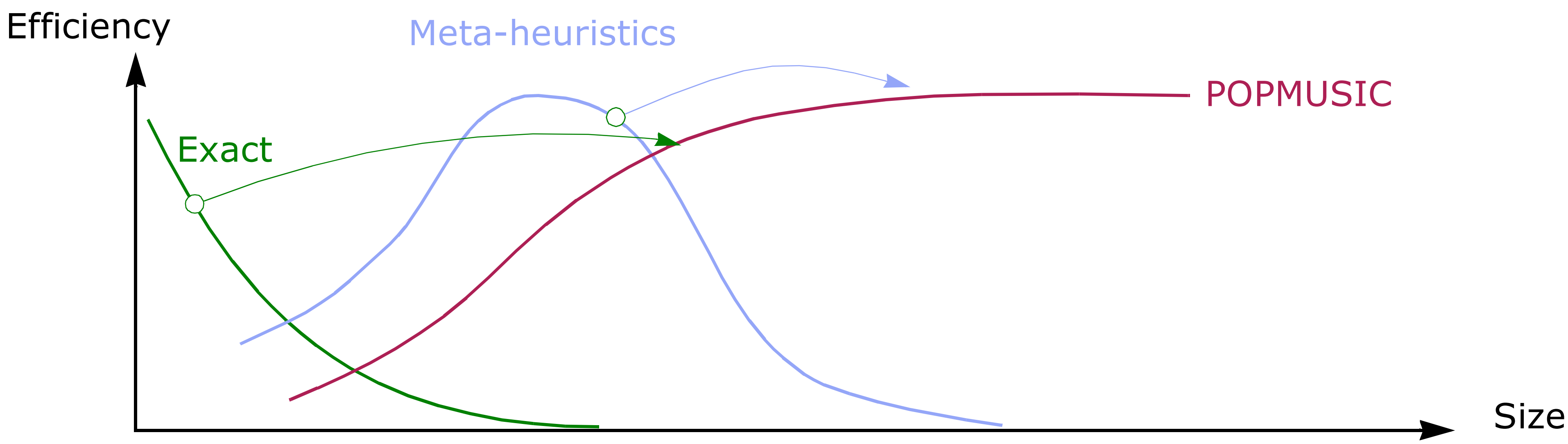
Distant elements are not directly connected together in reasonable solutions

Reasonable solutions are composed of sets with about C elements (independent of problem size)

Example : The number of letters a postman can deliver in a day does not depend on the total number of people living in the country

Classification of problem size

Class	Typical technique	Size (order)	
Toy	Complete enumeration	10^1	
Small	Exact method	10^1—10^2	
Medium	Meta-heuristics	10^2—10^4	Memory limit $O(n^2)$
Large	Decomposition techniques	10^3—10^7	Time limit $O(n^{3/2})$
Very Large	Distributed database	above	



Input

Solution $S = s_1 \cup s_2 \cup \dots \cup s_p$ // p disjoint parts

$U = \{ s_1, s_2, \dots, s_p \}$ // Set of “ un-optimized ” seed parts

While $U \neq \emptyset$, **repeat** // Parts that can still be used for creating sub-problems

1. Choose a seed part $s_i \notin O$

   // r : parameter

2. Create a sub-problem R composed of the r “ closest ” parts $\in S$ from s_i

3. Optimize sub-problem R

4. **If** R improved **then**

   Set $U \leftarrow U \setminus R \cup R^*$

   Else

   Set $U \leftarrow U \setminus s_i$

RELATED CONCEPTS

Candidate list, strongly determined and consistent variables (Glover)

“Chunking” (Woodruff)

Large neighbourhoods (Shaw)

Granular search (Toth & Vigo)

VDNS (Hansen & Mladenovic)

Matheuristics

Exchange (Pochet & Wolsey)

Fix-and-Optimize (Helber & Sahling)

POPMUSIC Algorithmic Complexity

Number of repetition of "While" loop

Empirically proportionnal to problem size

i.e. similar to local search, simplex pivoting

1. Choosing a seed part

Constant time

Random choice, Stack

2. Finding the *r* closest parts

Without specialized data structure : Proportionnal to problem size

3. Optimizing one sub-problem

Exponential with *r* but independent from problem size

In practice : *r* is fixed, so optimizing a sub-problem is performed in constant time

4. Updating *U* set

Proportionnal to *r*

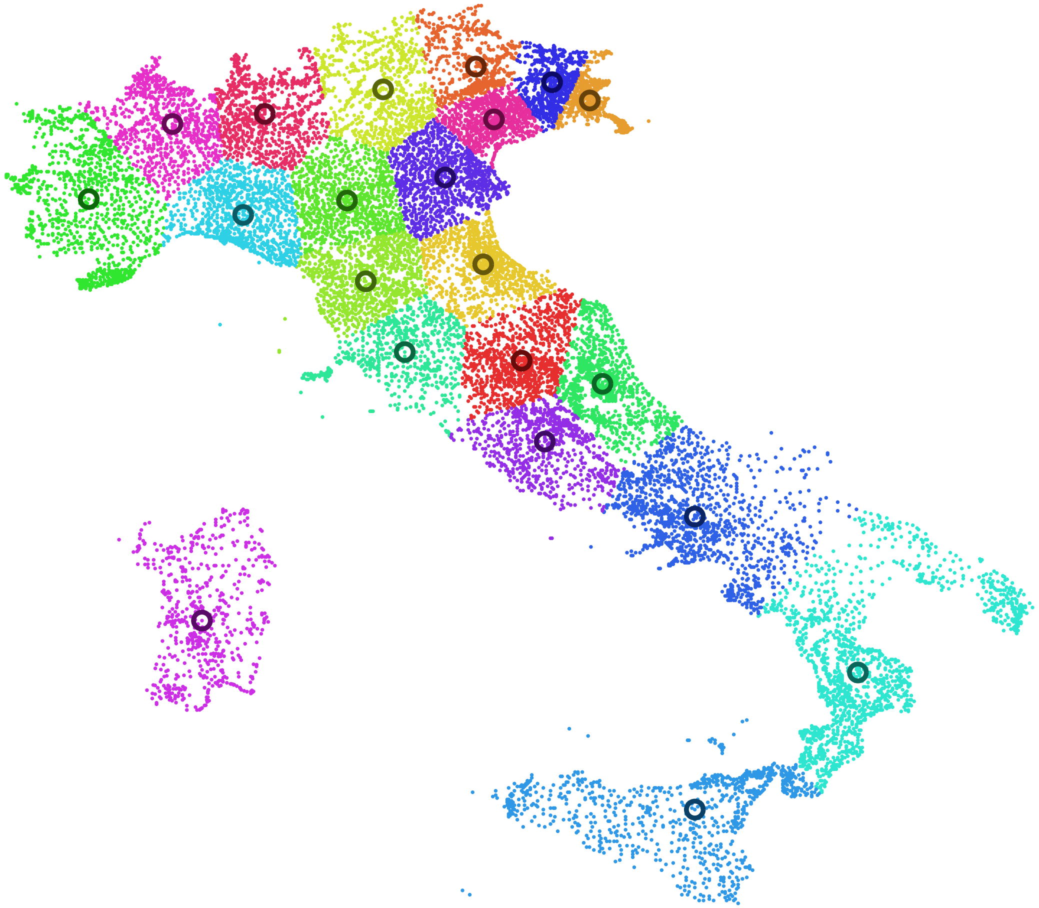
THE P-MEDIAN PROBLEM

Given :

n elements $\in I$ with distance matrix $D = (d_{ij})$ between them

Find :

p central elements $\{c_1, \ldots, c_p\} \in I$ minimizing $\sum_{i=1}^{n} min_{j=1,\ldots,p}(d_{i,c_j})$



Template for problem decomposition

Input

n elements, function $d(i, j)$ measuring the proximity between elements i and j

Body

1 Create a random sample E of $30\sqrt{n}$ elements

2 Solve a relaxation of a p-median with capacity with $p = \sqrt{n}$ on E

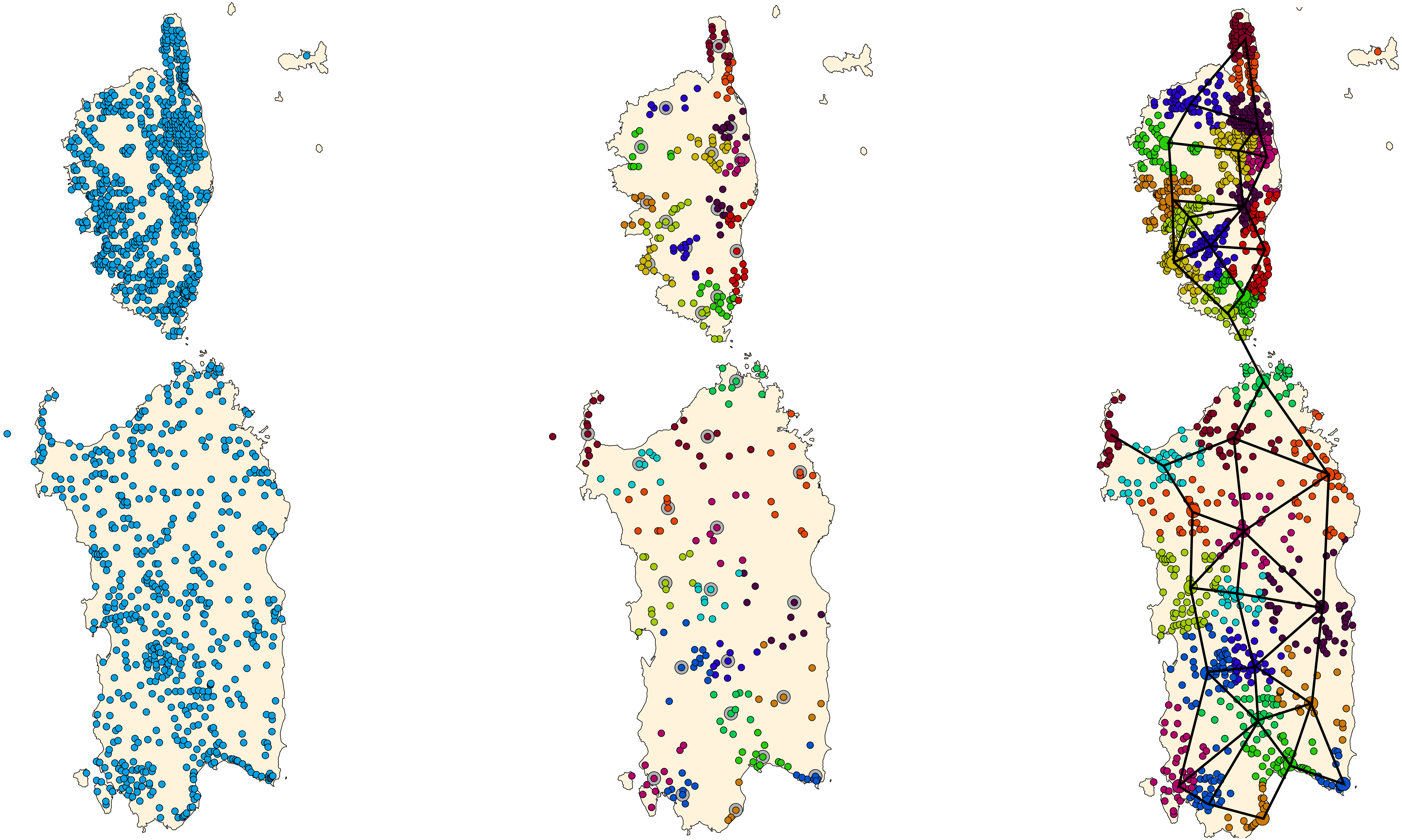
3 Assign each of the n elements to its closest among the p centres
$\Rightarrow \sqrt{n}$ clusters with $\sim\sqrt{n}$ elements each

4 Build a proximity graph G on the centres
$\Rightarrow c_i$ and c_j are neighbours if:
there is an element assigned to c_i which second closest centre is c_j

Output

$\sim\sqrt{n}$ clusters, proximity graph G

PROBLEM DECOMPOSITION



Initial set of elements

Sample + clustering

Assignement + proximity graph

Goal :

Decomposing a set $E = \{1, \ldots, n\}$ into p clusters $C_1, \ldots, C_p$ with $\sim n/p$ elements each

```
Randomly select a sample of e = min(n, 30·p) elements among the n entities
Randomly choose p centres c_1, …, c_p among the e entities
f = 0.6;
λ_j = 0; j = 1, …, p; // Penalty for each centre
for 30 iterations do
    for i = 1 to e do
        Allocate entity i to the centre j minimizing d(i,j) + λ_j
    for j = 1, …, p do
        Find the best position of centre c_j among entities assigned to it
    Compute solution cost C and store solution if best improved
    f ← 0.99 · f
    for j = 1, …, p do
        λ_j ← Max(0, λ_j + f · C · (n_j – e/p)·e^2)
        // n_j : number of entities allocated to centre j
for i = 1 to n do
    Allocate entity i to the centre j of best solution minimizing d(i,j)
```

Complexity

$$\Theta(e \cdot p + e^2 + n \cdot p) \Rightarrow \Theta(n^{3/2}) \text{ if } p \text{ in } \Theta(\sqrt{n})$$

Problem instances

From TSP library

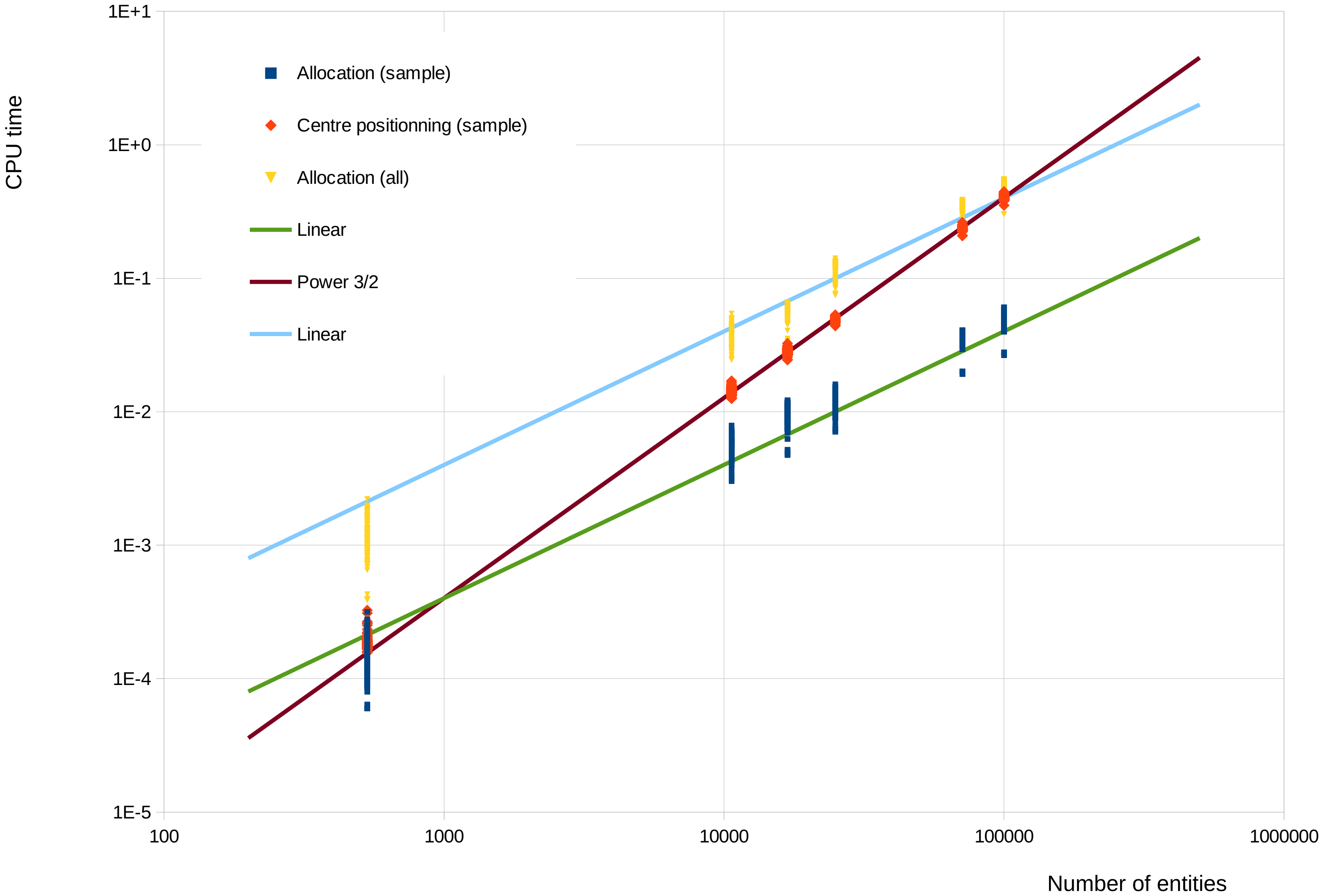
Att532	n = 532
Finland	n = 10639
Italy	n = 16862
Sweden	n = 24978
China	n = 71009
Mona-Lisa	n = 100000

Note :

Assuming Euclidean instances and using KD-Tree, k-means algorithm can be implemented much more efficiently

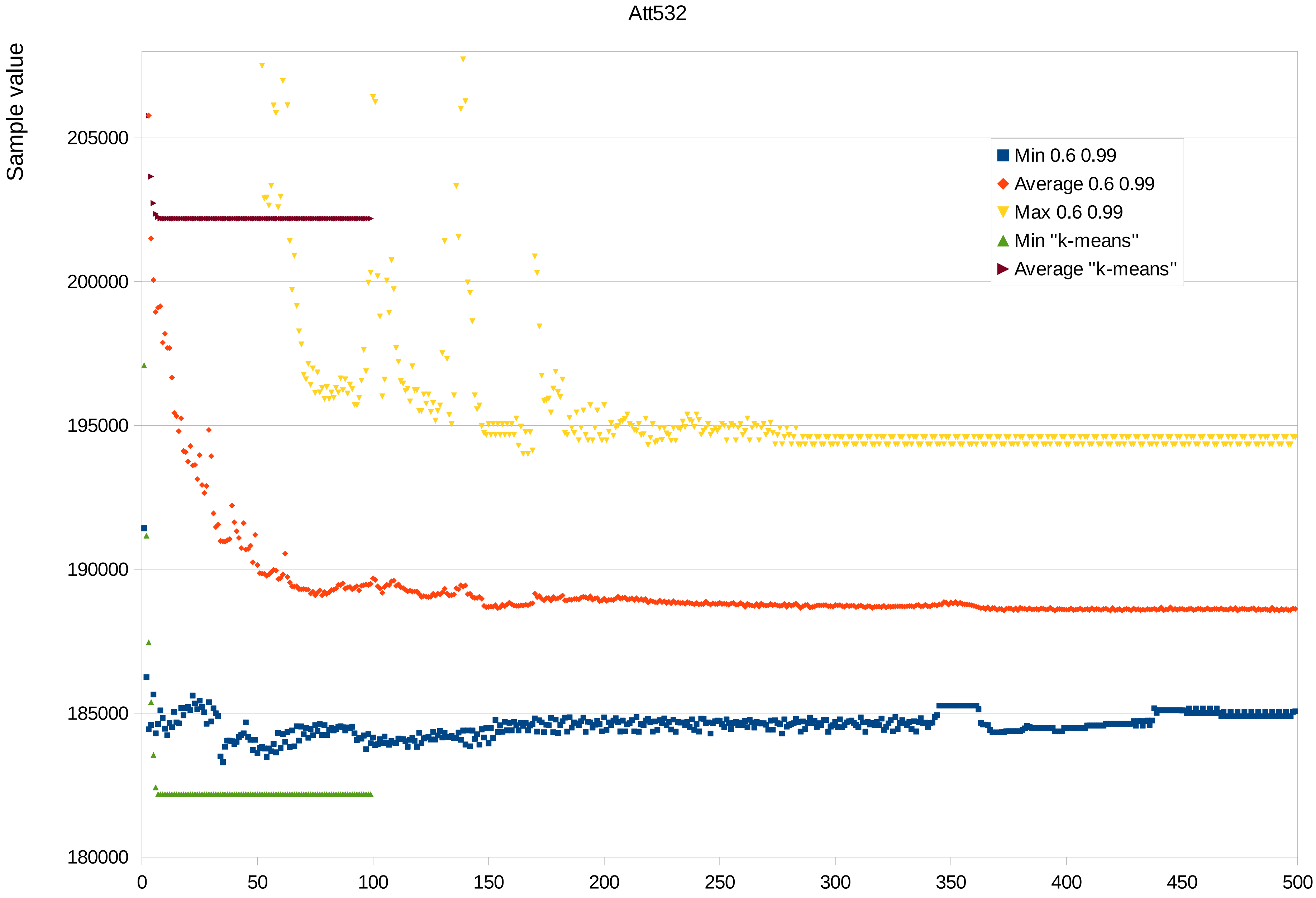
CPU Time

Seconds per iteration on intel i7 930 2.6GHz

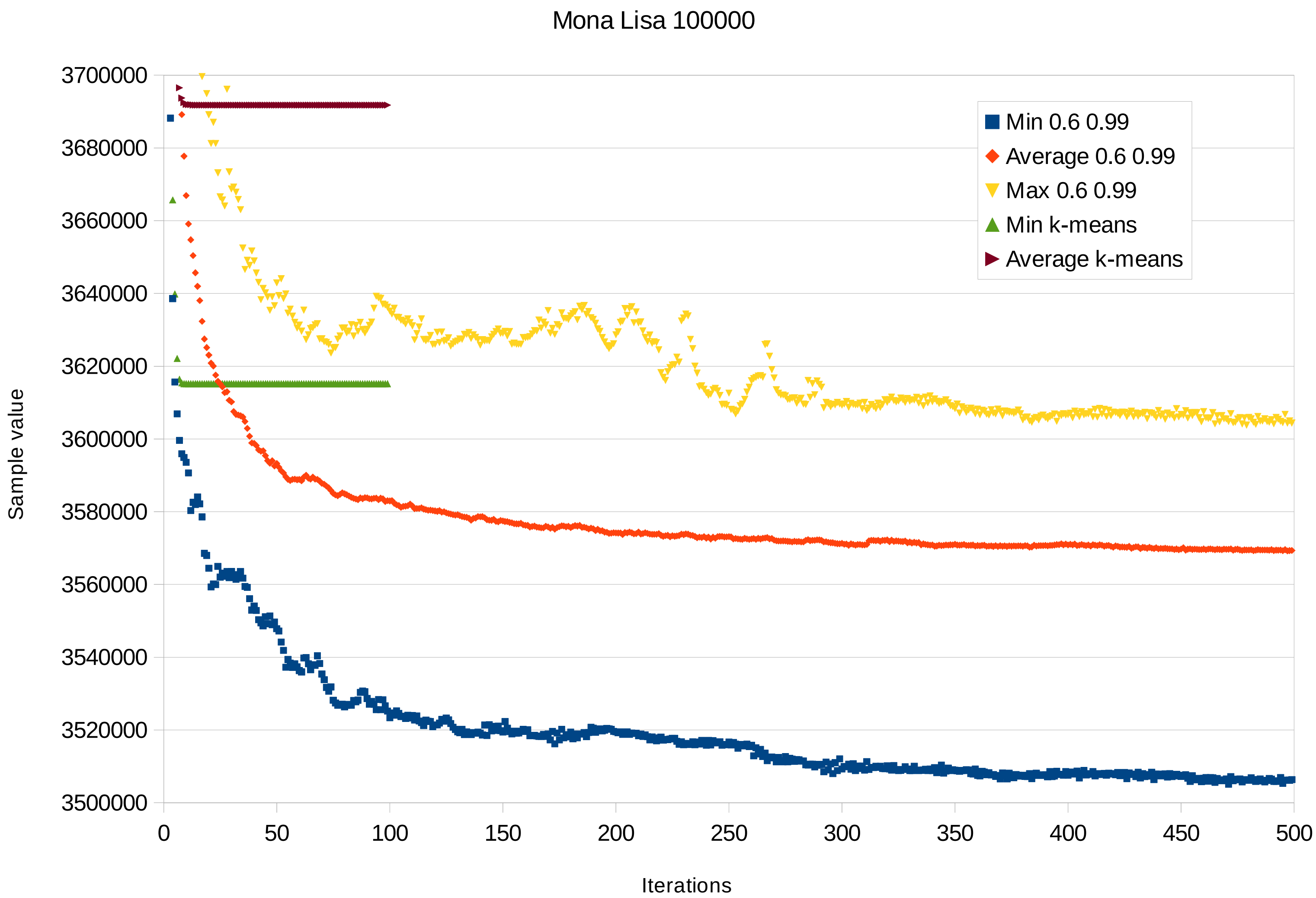


Solution quality

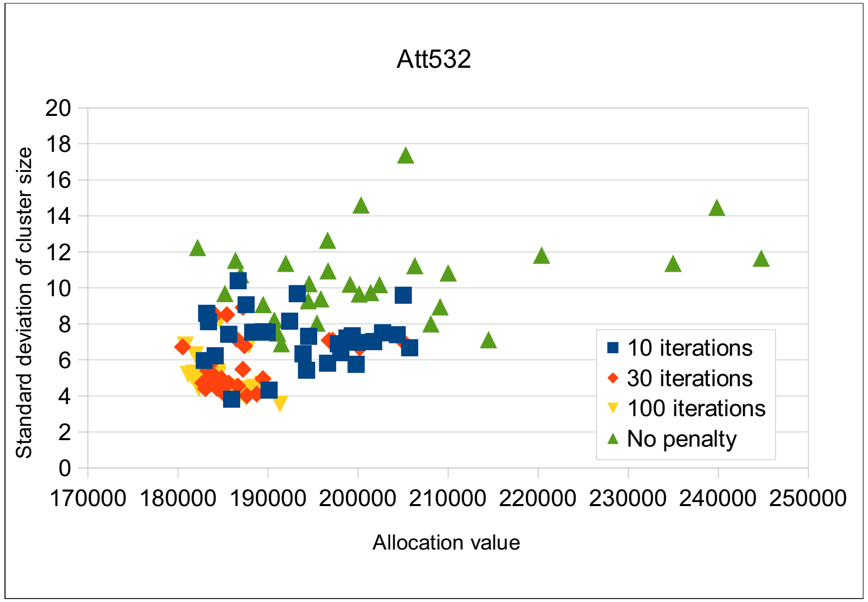
Comparison with k-median heuristic, 30 independent runs

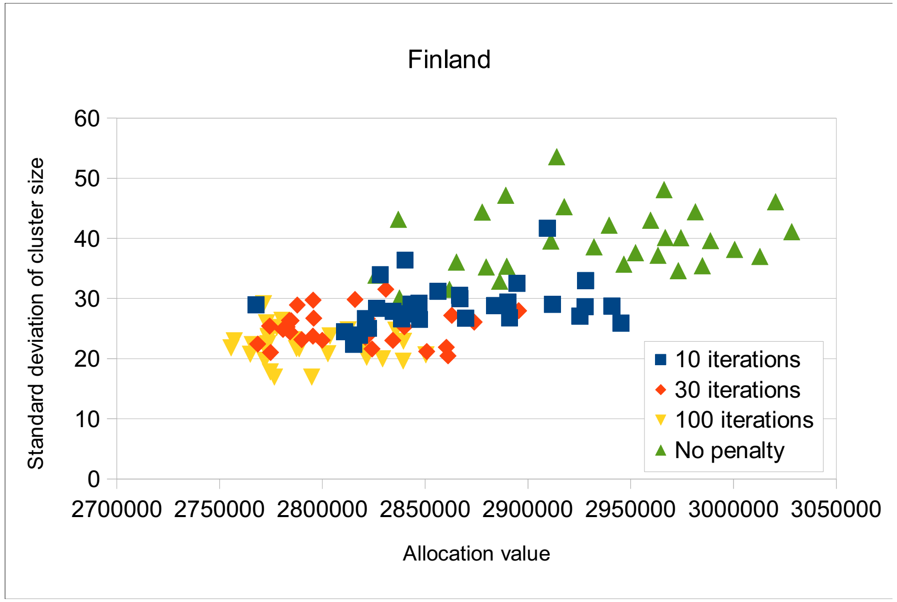


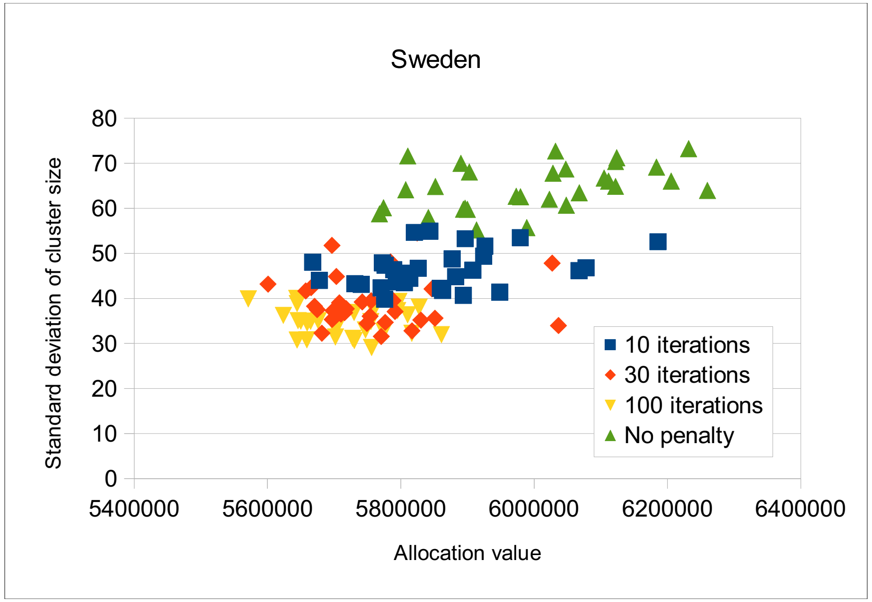
Mona Lisa, 30 independent runs

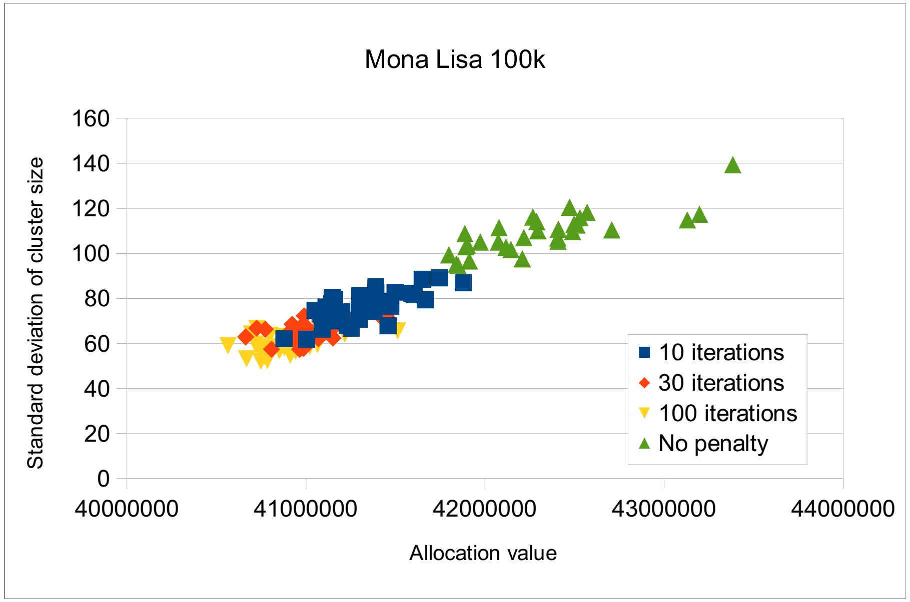


Standard deviation of cluster size



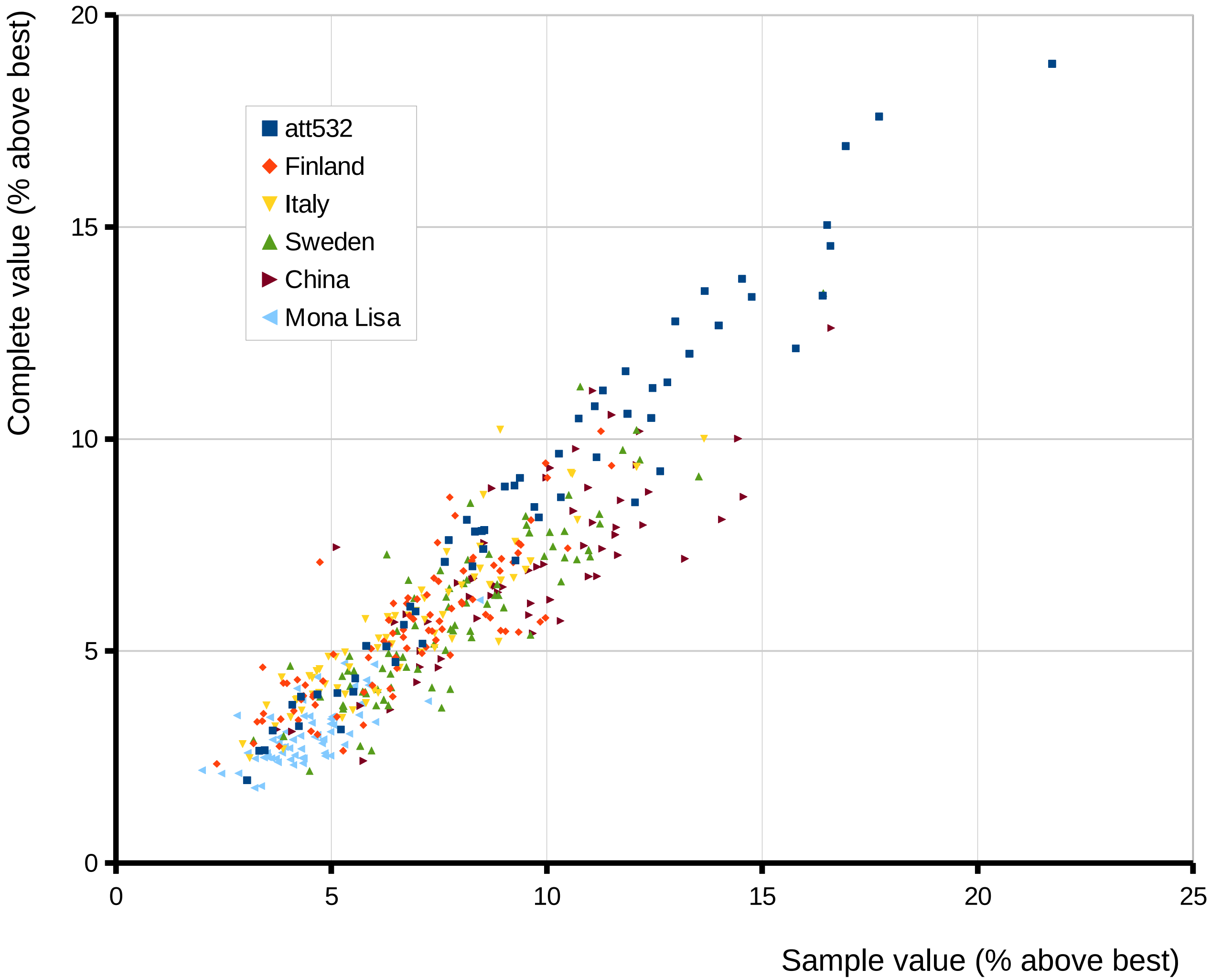




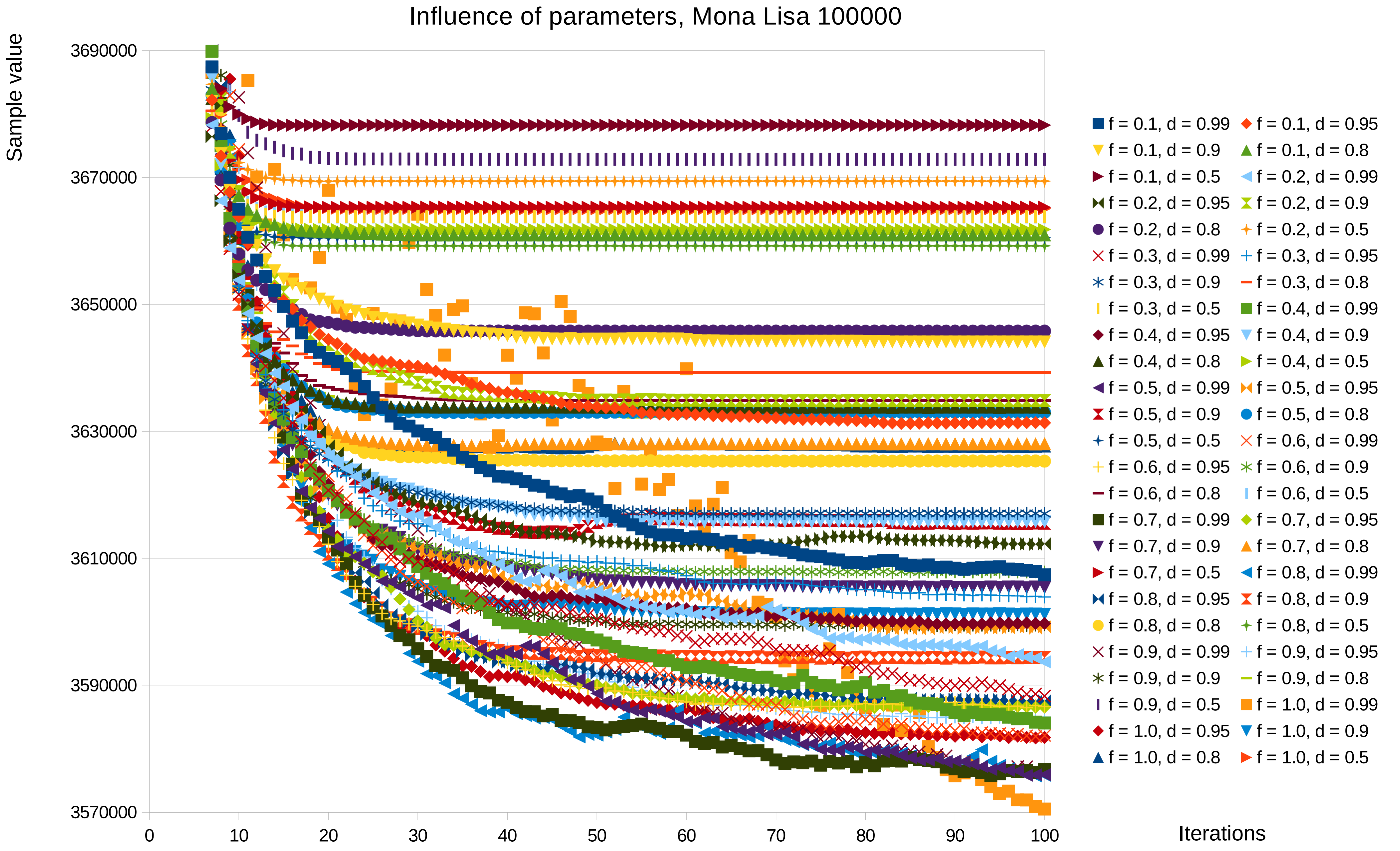


CORRELATION BETWEEN SAMPLE AND COMPLETE COST

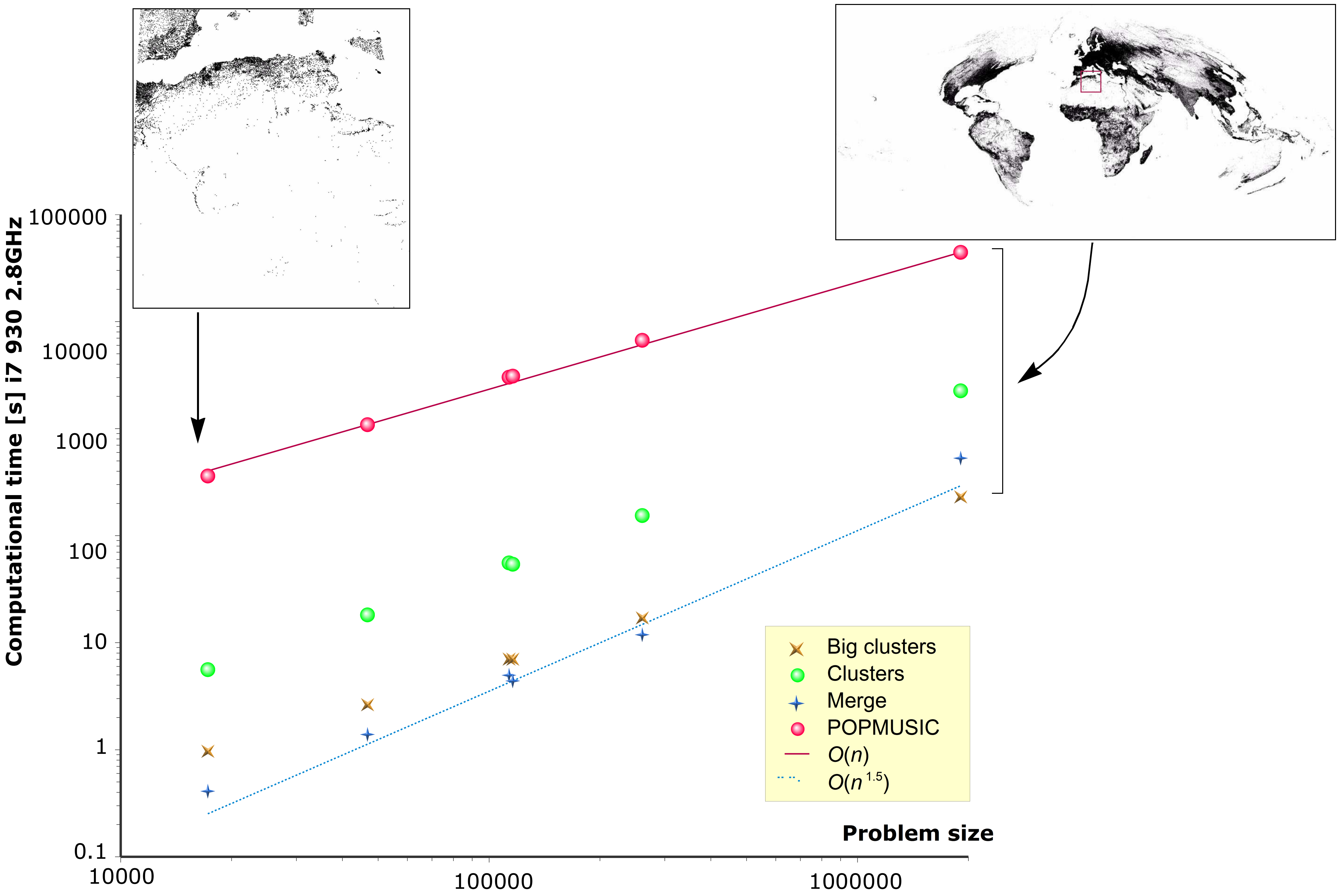
Reference : best solution found during all runs



Parameter Tuning



Application to Location-Routing : Empirical algorithmic complexity



CONCLUSIONS

POPMUSIC complexity

Can be implemented in $O(n^{3/2})$

Main difficulty : generating an initial solution, finding the *r* closest parts

$\Rightarrow$ Solved with proximity graph

POPMUSIC options and parameter

Natural stopping criterion

Must have an optimization process for sub-problems

Heuristic

Exact $\Rightarrow$ Matheuristic

A single parameter *r*, for defining sub-problem size

$\Rightarrow$ Easy to tune : sub-problem size must meet best efficiency of optimization process

POPMUSIC drawback

Definition of part and sub-problem dependent on problem under consideration

FUTURE

Application to higher dimensional instances

Up to now : Map labelling 2D, Location-routing $2^{1/2}$D, MDVRPTW 3D

What happens for higher dimensions ?

Application to other problems

Testing different definitions for parts

Study of different options

Definition of distance between parts

Management of non-optimized parts

Building different proximity graphs

Parallel implementations