An *n log n* Heuristic for the TSP

Éric D. Taillard

Cartagna, Columbia, July 2019

TSP Stat of the Art

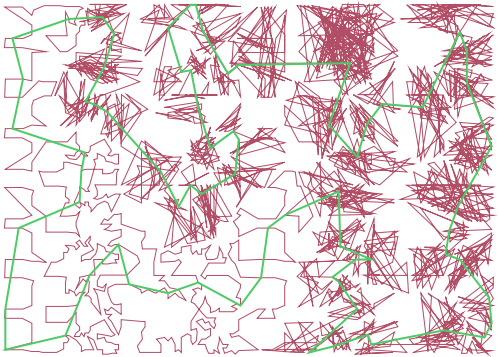
- "Exact" algorithms
  - Concorde (Applegate, Bixby, Chvátal, Cook, 2003 [1])
  - Optimum known for $\approx$ 100k cities instances
- Heuristics
  - Local search with Lin & Kernighan (1973) [9] neighbourhood
    + k-opt moves(algorithmic complexity: $k = 5 \implies O(n^{4 \cdot \cdot})$)
    + neighbourhood reduction (keep ≈ 5 adjacent edges for each city)
    + iterated random perturbations (double bridge)
  - One of the best algorithms due to K. Helsgaun (LKH, 2000–2018 [4, 5, 3, 7, 8])
  - Solutions less than 0.0...% above optimum for all instances of the litterature (up to 10^7 cities)

Tours merging strategy

- Generate rapidly several different, moderately good tours (few dozens)
- Build a partial graph containing only the edges of these tours
- Apply a local search limited to the edges of the partial graph
- - Known process, allowing to get good solutions (Blazinskas et Misevicius (2012) [2])
- Difficulty: How to get moderately good solutions for non Euclidean instances with a limited complexity ($< O(n^2)$)

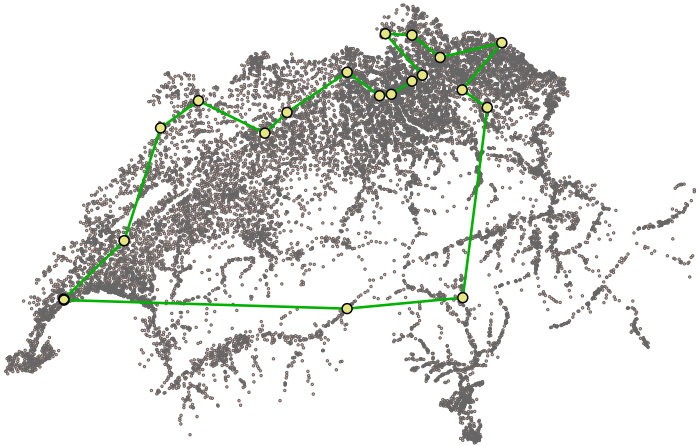
Method in $O(n^{1.57})$ (MIC 2017, EJOR 2019 [10, 11])

- Select a sample of $O(n^{0.56})$ cities
- Find a good tour on the sample with Lin-Kernighan neighbourhood
- Group all the cities into a number of clusters equals to the sample
- Optimize the tour with 2-opt neighbourhood by considering 2 successive clusters at a time
- Re-optimize the tour with POPMUSIC (all subsets of 50 successive cities are local optimum)



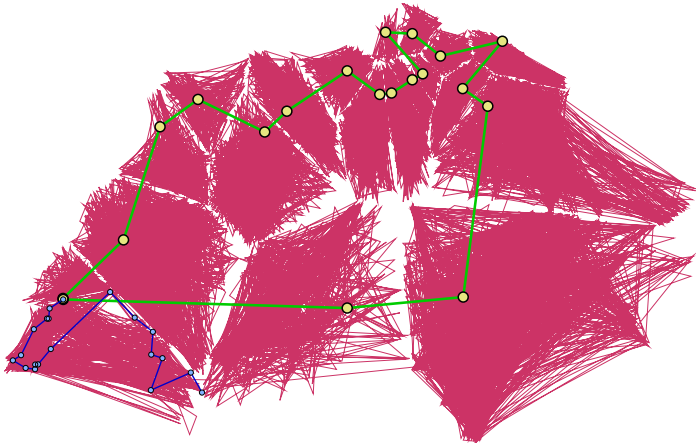
New Method in $n \log n$

- Recursive decomposition with a fixed sample size t



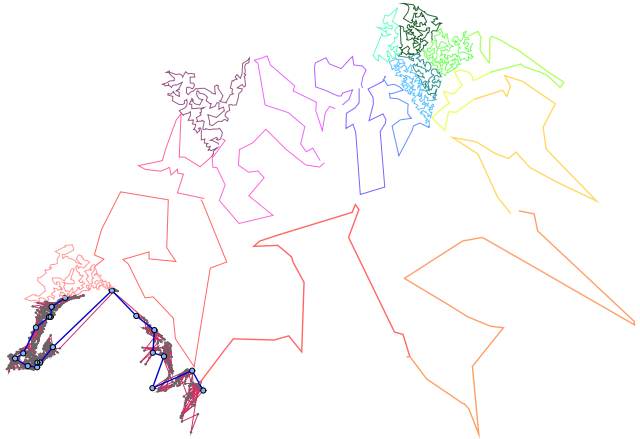
First complete solution obtained in linear time

- Clusters that are larger than t^2 are decomposed into t clusters

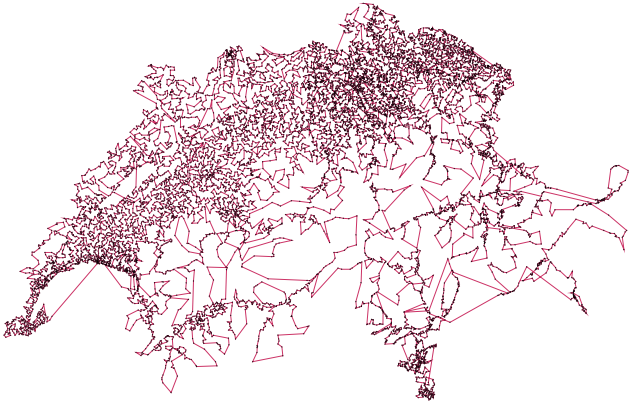


Iterate until cluster size $< t^2$

- A good path for a cluster with few hundreds of cities can be rapidly obtained with Lin-Kernighan neighbourhood

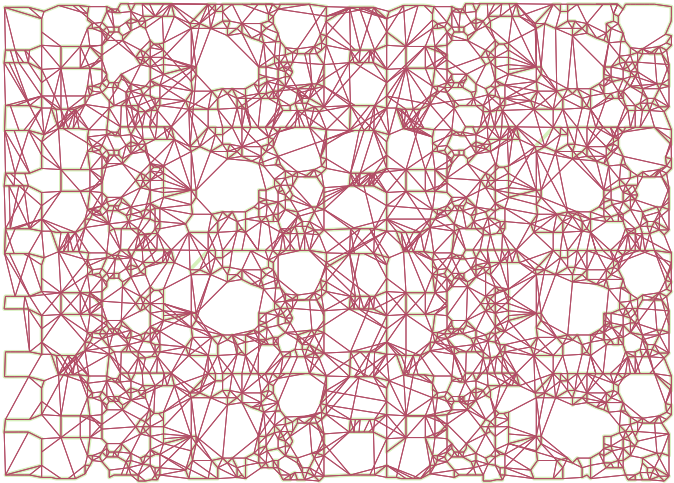


Solution finally built

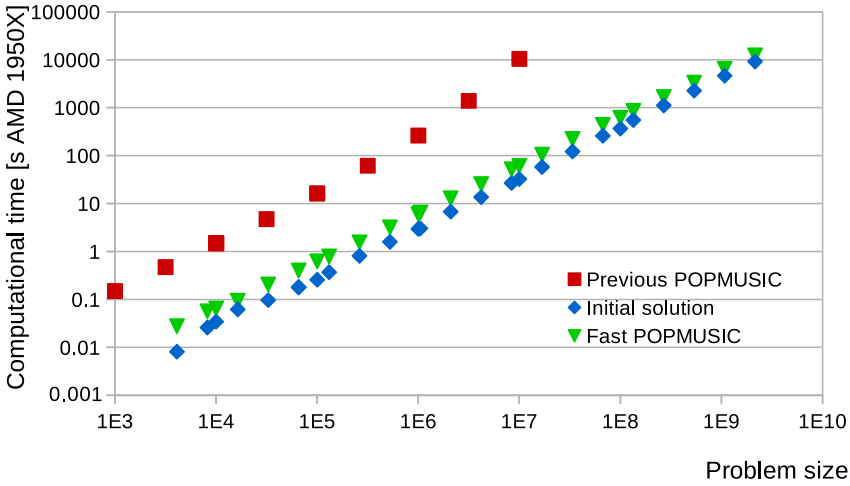


- The solution is improved with a fast POPMUSIC
  - Non-overlapping paths with t^2 cities are independently optimized
  - Only 2 scans, shifted by $t^2/2$ cities

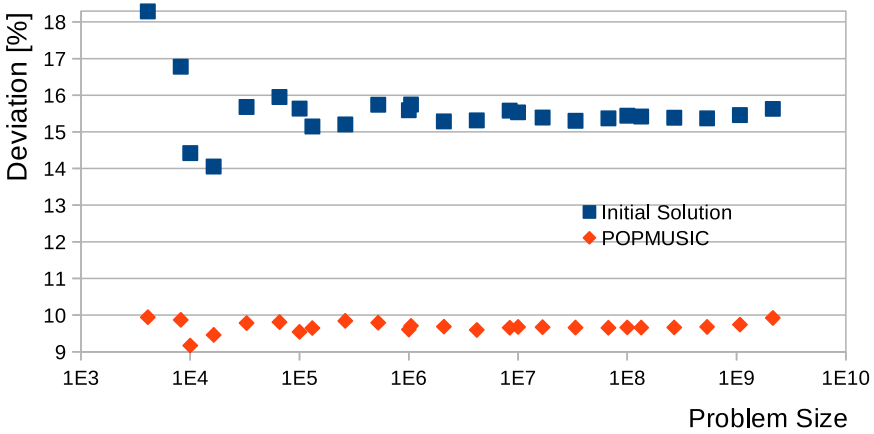
Union 20 solutions superimposed to an optimal solution



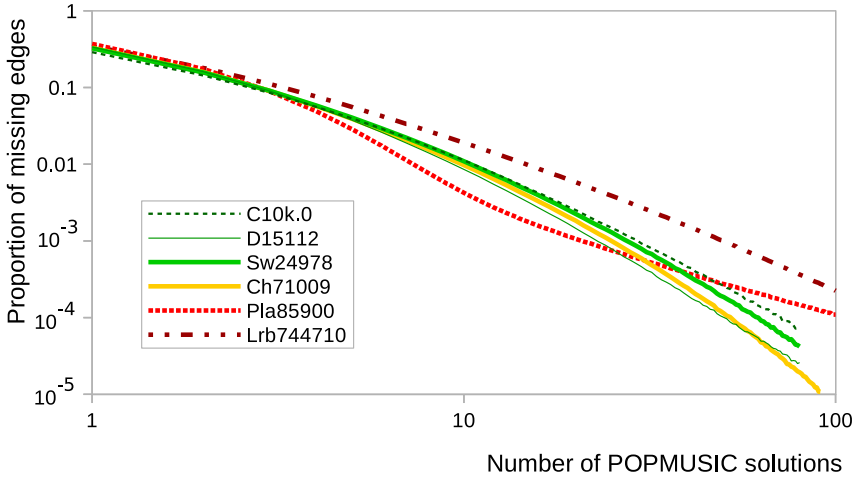
Computational time as a function of problem size



Solution quality for 2D toroidal instances

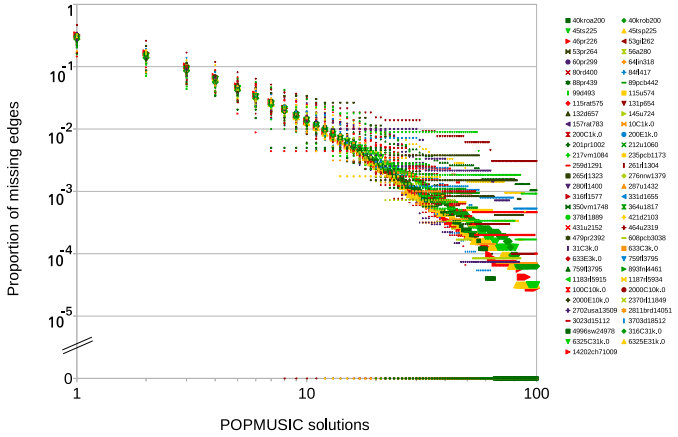


Proportion of missing edges relatively to best solutions



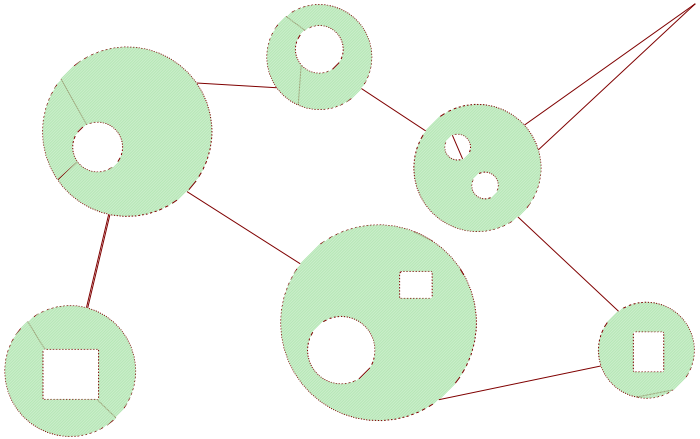
Application to the Clustered TSP

Proportion of edges missing from best solutions (Helsgaun 2014 [6])



Application to 3D extrusion additive manufacturing

Minimizing unproductive moves



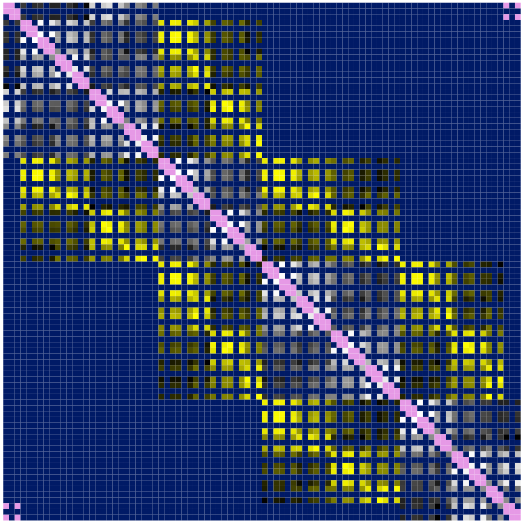
3D additive manufacturing: TSP with special dist. matrix

Big M distance

Distance + M/10

Distance proportionnal to darkness

Distance = 0



Application of LKH to 3D printing

- Difference (%) of unproductive length between LKH-2.0.7 and new version with POPMUSIC candidate edges
- Head cleaning every 2 layers

Instance	1	2	3	4	5	6	7	8
Size	4338	5427	4503	3669	13359	6606	6066	10872
Difference	-0.4	0.6	0.7	0.7	-0.8	0.1	1.4	0.9

- Comparison with specially designed methods 1 & 2 of (Volpato et al. 2019 [12]) on instances with cleaning every 2 layers
- Measure : Improvement (%) relatively to solutions of methods 1 & 2

Instance	1	2	3	4	5	6	7	8
Size	2574	2997	3159	3516	6696	4122	6834	6513
Method 1	8.8	30.2	24.0	4.6	38.9	20.8	24.3	11.7
Method 2	6.9	24.6	22.4	1.4	29.7	11.4	15.6	9.6

Conclusions

- New $n \log n$ method providing an excellent edge filtering for the TSP
  - Euclidean instances
  - 2D, 3D, 4D Toroidal distances
  - Clustered TSP
  - Special distances for 3D Printing
- Direct application of the method
  - Large instances that must be solved with a limited computational power
  - Drone survey
- How to filter data for other problems ?
  - Seems not evident for the QAP

David L. Applegate, Robert E. Bixby, Vasek Chvátal, and William J. Cook.
Concorde: A code for solving traveling salesman problems, 1999.

Andrius Blazinskas and Alfonsas Misevicius.
Generating high quality candidate sets by tour merging for the traveling salesman problem.
In Tomas Skersys, Rimantas Butleris, and Rita Butkiene, editors, *Information and Software Technologies: 18th International Conference Proceedings, ICIST 2012, Kaunas, Lithuania, September 13-14, 2012*, pages 62–73. Springer, Berlin, Heidelberg, 2012.

Keld Helsgaun.
Best known solutions to Dimacs TSP instances.
Last updated: October 6, 2014.

Keld Helsgaun.
An effective implementation of the Lin-Kernighan traveling salesman heuristic.
European Journal of Operational Research, 126(1):106–130, 2000.

Keld Helsgaun.
General k-opt submoves for the lin-kernighan tsp heuristic.

Mathematical Programming Computation, 1, 2009.


Keld Helsgaun.
Solving the clustered traveling salesman problem using the lin-kernighan-helsgaun algorithm.
Technical report, Department of Computer Science, Roskilde University, DK-4000 Roskilde, Denmark, 2014.


Keld Helsgaun.
Helsgaun's implementation of Lin-Kernighan, 2016.
Version LKH-2.0.7.


Keld Helsgaun.
Using popmusic for candidate set generation in the lin-kernighan-helsgaun tsp solver.
Technical report, Department of Computer Science, Roskilde University, DK-4000 Roskilde, Denmark, 2018.


S. Lin and B. W. Kernighan.
An effective heuristic algorithm for the traveling-salesman problem.

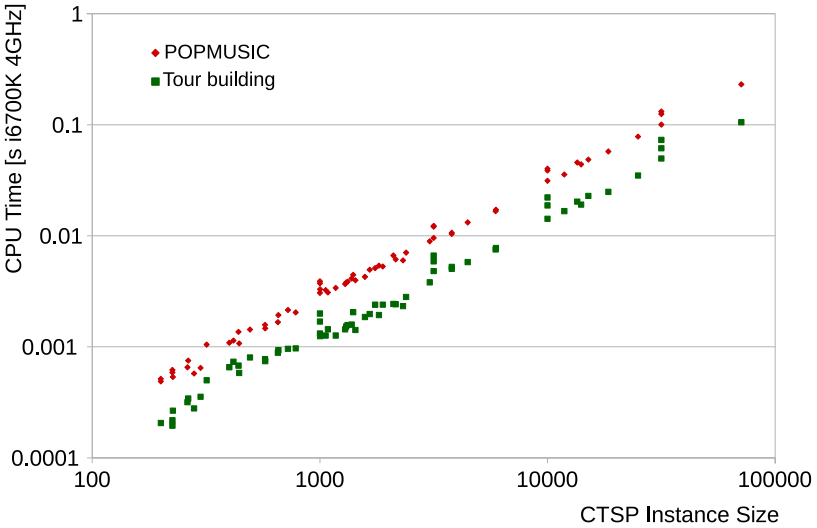
Operations Research, 21(2):498–516, 1973.


Éric D. Taillard.
TSP neighbourhood reduction with POPMUSIC.
In *Metaheuristic Interantional Conference (MIC'17) proceedings*, 2017.


Éric D. Taillard and Keld Heslgaun.
POPMUSIC for the travelling salesman problem.
EURO Journal of Operational Research, 272(2):420–429, 2019.


Neri Volpato, Lauro Cesar Galvão, Luiz Fernando Nunes, Rômulo Ianuch Souza, and Karina Oguido.
Combining heuristics for tool-path optimisation in material extrusion additive manufacturing.
Journal of the Operational Research Society, ?(On-line):1–11, 2019.

Application to the Clustered TSP



3D additive manufacturing: Comparing solutions Part4